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On Experimental Verification of Einstein's Equivalence Principle and Space-Time Coordinate Systems in General Relativity

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Abstract: Einstein's equivalence principle, which states the local equivalence between gravity and acceleration with respect to a frame of reference, requires that a free falling observer result in a co-moving local Minkowski space as a spaceship under only the influence of gravity. However, a "free fall" in a Lorentz manifold may not uniquely result in a local Minkowski space, and a Lorentz manifold with a non-constant metric need not imply the existence of static acceleration. Thus, Einstein's equivalence principle, different from Pauli's version, requires that the physics related to the local coordinate transformation must be proven valid. It is shown that Einstein's equivalence principle restricts the valid space-time coordinate systems to ensure Einstein's Riemannian space-time to be a physical space. Experimentally, the light bending supports the Euclidean-like structure of the frame of refer-ence, and the gravitational redshifts support that time is defined "at the rate of a clock depends upon where the clock may be." Moreover, the observed gravitational redshifts illustrate that physically valid diffeomorphisms and space-time coordinate sys-tems are restricted by Einstein's equivalence principle. Thus, Pauli's version of equivalence principle is inadequate since some Lorentz manifolds are not valid in physics and diffeomorphic manifolds may not be equivalent. It is interesting to note that the earliest experimental support of Einstein's equivalence principle would be the Michelson-Morley experiment.
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1. Introduction.

Einstein's equivalence principle requires that a free fall in a physical space is along a geodesic and results in a co-moving local Minkowski space[1].[1,2] Some theorists including Pauli [3-14], however, believed the reverse that a mathematical existence of local Minkowski spaces would be sufficient for a satisfaction of Einstein's equivalence principle in a manifold although Einstein objected this misinterpretation (see Appendix). The acceptance of Pauli's misinterpretation is facilitated by its consistence with Einstein's interim proposal [2] that an arbitrary Gaussian system can be a coordinate system, and the readily application of mathematical theorems that a Lorentzian metric signature [14]

[1] A local Minkowski space is a short hand to express that special relativity is locally valid in the co-moving local Minkowski space, except for phenomena involving the space-time curvature.

implies the mathematical existence of local Minkowski spaces. However, a problem is that physical requirements beyond metric signature are ignored.

Thus, it is not surprising that a major problem in current theories is that many Lorentz manifolds are not valid in physics although they are solutions of Einstein's field equation.[15] Also, unrestricted covariance has been found to be a problem for a consistent physical interpretation of the space-time metrics.[16,17] Thus, one may question whether a satisfaction of the equivalence principle is sufficient for a manifold to be a physical space; or consider the possibility that a mathematical existence of local Minkowski spaces is necessary but insufficient for a satisfaction of the equivalence principle.[18-20]

Einstein [1,2] proposed his equivalence principle for a physical space, which models reality for physical problems. Einstein's physical space, where physical requirements are sufficiently satisfied, includes a frame of reference and a time coordinate such that the time-rate and the-time dilation determine the rate of a local clock.[1,2] Thus, Einstein's physical space is restricted by physical requirements on the coordinate system. Since the transformed manifold may not have a frame of reference and a time-coordinate related to local clocks as physics requires, diffeomorphic manifolds may not be equivalent in physics.

Einstein's equivalence principle is applicable to the special case, the Minkowski metric. Thus, the earliest experiment that shows two diffeomorphic manifolds are not equivalent[2] is the Michelson-Morley experiment [21], which attempted to detect the then expected differences in directional local light speeds. This experiment found that a Galilean transformation, which is a diffeomorphism, is not realizable since the so transformed coordinate system does not exist in reality. However, theorists were so overwhelmed by the observational confirmation of Einstein's predictions, except outstanding scientists such as Gullstrand [22,23], Eddington [24], and Pauli [5], few paid attention to examine carefully the theoretical foundation of general relativity.

There are many experimental evidences against physical equivalence of diffeomorphic coordinate systems, because of the existence of non-scalars in physics. However, many current theorists ignored them by claiming that coordinates have no physical meaning.[6] Thus, they also had to dismiss that Einstein's coordinate light speed as having no physical meaning. Then, the light speed was dubiously defined in the local Minkowski spaces [19] although such a definition can hardly be related to the bending of light. Of course, it does not occur to those theorists [6] that a Lorentz manifold may not be diffeomorphic to a physical space.[20] In other words, without any physical evidence, some relativists [6] incorrectly believe general covariance of physical laws beyond the principle of general relativity and the equivalence principle.

In this paper, it will be shown that Einstein's equivalence principle implies the existence of non-equivalence among diffeomorphic coordinate systems. It has been established in special relativity that the Minkowski metric restricts a space-time coordinate system. In general relativity, this role of restriction is played by Einstein's

[2] For two diffeomorphic manifolds, if they are not equivalent, at least one of them is not a physical space, and thus also not realizable. Then, experimentally, one can show only its absence. The fact that some theorists do not understand special relativity adequately, however, is not an isolated incident (please see also Appendix).

equivalence principle. Noted experimental evidences for such a restriction includes the observed gravitational redshifts and the bending of light rays. However, such a restriction was not clearly understood because the physical meaning of coordinates was ambiguous.[25-27] Fortunately, it is found that, in terms of the Euclidean-like structure, the space-time coordinates have definite physical meanings (see Section 3).

2. Einstein's Equivalence Principle, Special Relativity, and the Gravitational Red Shifts

In 1911, Einstein [21] derived the gravitational red shifts directly from the initial form of his equivalence principle, i.e., the equivalence of an accelerated frame of reference and *uniform* gravity. This work is important because such a direct relationship is a basis of Einstein's confidence on his later derivation [2] on the bending of light rays.

In a gravitational field all particles fall with the same acceleration. Thus the passive gravitational mass and inertial mass are considered as equivalent (the weak equivalence principle). Einstein assumed that the mechanical equivalence of an inertial system K under a uniform gravitational field, which generates a gravitational acceleration γ (but, system K is free from acceleration), and a system K' accelerated by γ in the opposite direction, can be extended to other physical processes.

Consider two material systems S_1 and S_2 which are situated initially at rest on the z-axis of system K and are separated by a distance h so the gravitation potential in S_2 is greater that S_1 by γh. If a definite radiation energy E_2 be emitted from S_2 to S_1 at the moment that system K' has zero velocity relative to an inertial system K_0, the radiation will arrive at S_1 when the time h/c has elapsed (to a first order approximation); and at this moment the velocity of S_1 relative to K_0 is $\gamma h/c = v$. *According to special relativity*, the radiation arrives S_1 with a greater energy E_1 which (to a first order approximation) is related to E_2 by

$$E_1 = E_2(1 + v/c) = E_2(1 + \gamma h/c^2) \qquad (1)$$

By assumption, exactly the same relation holds if the same process takes place in the system K, which is not accelerated, but is provided with a gravitational field. Then, we may replace γh by the gravitational potential Φ and obtain

$$E_1 = E_2(1 + \Phi/c^2) = E_2 + \Phi(E_2/c^2) \quad . \qquad (2)$$

Thus, the energy increment of radiation due to gravity is resolved by the equivalence of the K and K' systems. *The omitted step of replacing γh by $\Delta\Phi$, though obvious, is crucial since a small enough $\Delta\Phi$ can be considered as due to uniform gravity*[3)]. Einstein's

[3] A uniform acceleration cannot exist forever; otherwise the resulting speed would exceed the speed of light. Thus, a uniform acceleration must be started and then decreased some time afterward, and must be confined in a finite region. In practice, uniform gravity is an idealization, like a plane wave, that is useful in dealing with a local problem.

derivation of (2) could be misinterpreted as that Einstein regards any gravity is equivalent to acceleration.[28]

If the radiation emitted in system K' in S_2 towards S_1 had the frequency ν_2 relatively to the clock in S_2, then at the arrival of radiation in S_1, it has a great frequency ν_1 relatively to S_1, such that to a first approximation $\nu_1 = \nu_2(1 + \gamma h/c^2)$. If the radiation is emitted at time that K' has no velocity, S_1 at the time of arrival of the radiation, has relative to K, the velocity $\gamma h/c$. This is an immediate result of the Doppler's principle. If γh is substituted by the gravitational potential Φ of S_2 - that of S_1 being taking as zero - then the equivalence principle, to the first order approximation gives

$$\nu_1 = \nu_2(1 + \Phi / c^2) \quad . \tag{3}$$

If on the surface of a star (where S_2 is located) the light is emitted to the Earth (S_1) where the frequency of the arriving light is measured, then eq. (3) implies $\nu = \nu_0(1 + \Phi/c^2)$, where Φ is the (negative) difference of gravitational potential between the surface of the star and the Earth. Formula (3) is also confirmed in 1916 by Einstein's result of time dilation [2].

The derivation of redshift without the notion of a curved space is valid due to that a curved space has the Euclidean-like structure (Section 3). It is interesting to note that the space coordinates of the Euclidean-like structure are essentially as in special relativity. However, in deriving the light speeds (related to $ds^2 = 0$), it is invalid to regard the space having a Euclidean subspace.

3. Einstein's Equivalence Principle, Free Falling, and Special Relativity

The discovery of a Riemannian space-time is due to the principle of general relativity. Mathematics shows that a free falling observer necessarily results in a locally constant metric. To be consistent with the initial form of his equivalence principle, Einstein [2] proposed, as an integral part of his equivalence principle, "special theory of relativity applies to the special case of the absence of a gravitational field." Thus, to understand Einstein's equivalence principle, one must first understand why the Minkowski metric is unique because of special relativity.[21]

On the other hand, Einstein's equivalence principle is necessary by observation. For example, when a spaceship is under the influence of gravity only, the local space-time is automatically Minkowski by the physics of gravity, but there is no other possible choice. Moreover, special relativity is naturally a special case since a Lorentz transformation results in a Minkowski space.

It should be noted that a free falling observer automatically results in a local Minkowski space is a physical requirement of Einstein's equivalence principle. However, in a Lorentz manifold, a "free falling" may not result in a co-moving local Minkowski space.[19,20] Such a transformation becomes automatic only if there is a physical cause, i.e., gravity (or relative velocity) to force such a process. If such a forcing cause does not exist, the principle of causality[4] is violated. Thus, a crucial problem for a Lorentz manifold is whether the time-like geodesic represents a free fall in a physical space.

Einstein clarified in his book [1], "According to the principle of equivalence, the metrical relation of the Euclidean geometry are valid relative to a Cartesian system of reference of infinitely small dimensions, and in a suitable state of motion (free falling, and without rotation)." Thus, a satisfaction of his principle can be decided by results so derived. Now, at any point (x, y, z, t) a "free falling" observer P must be in a co-moving local Minkowski space L with the local metric

$$ds^2 = c^2dT^2 - dX^2 - dY^2 - dZ^2 , \qquad (4)$$

whose spatial coordinates are attached to P, whose motion is governed by the geodesic,

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0, \text{ where } ds^2 = g_{\mu\nu}dx^\mu dx^\nu \qquad (5)$$

and

$$\Gamma^\mu{}_{\alpha\beta} = (\partial_\alpha g_{\nu\beta} * \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})g^{\mu\nu} / 2 \quad ,$$

are defined by the space-time metric $g_{\mu\nu}$. There is no relative motion, nor relative acceleration between P and L. (Note that the local space being Minkowski is only a mathematical choice in Pauli's versions of equivalence principle [5].) When gravity (i.e., gravitational acceleration) exists, correspondence with Newton's theory requires $\Gamma^\mu{}_{tt} \neq 0$ ($\mu \neq t$), and thus the space-time metric $g_{\mu\nu}$ is not a constant. The frame of reference attached to L is (X, Y, Z) is under acceleration.

On the other hand, for a given non-constant metric $g_{\mu\nu}$, acceleration may not exist for a static observer, i.e., $\Gamma^\mu{}_{tt} = 0$ ($\mu \neq t$). However, since there is no acceleration or motion for an initially static particle, it remains in the same position with the same frame of reference. Although the metric at a point be transformed to a local Minkowski space, is mathematically possible, such a transformation would lead to *inconsistent predictions* in physics because the metric is non-constant. Thus, Einstein's equivalence principle removes the possibility of gravity without any static acceleration.

To illustrate the above analysis, let us consider a Lorentz manifold with the non-constant metric,

$$ds^2 = c^2ch^2(T/C)dT^2 - dx^2 - dy^2 - dz^2, \qquad (6)$$

[4] The time-tested assumption that phenomena can be explained in terms of identifiable causes is called the principle of causality. This principle implies that any parameter in a physical solution must be related to physical causes [20,30,34]. In general relativity, Einstein and subsequent theorists have used this principle implicitly on symmetry considerations [1-14].

where C is a constant. From metric (6), the Christoffel symbols are zeros except $\Gamma_{t,tt} = \partial_t g_{tt}/2$, and thus there is no acceleration in eq. (5). The equation of motion for an observer P at (x_0,y_0,z_0,T_0) would be

$$\frac{d^2T}{ds^2}+\Gamma^t{}_{tt}\frac{dT}{ds}\frac{dT}{ds} = 0, \quad \text{and} \qquad \frac{d^2x}{ds^2}=\frac{d^2y}{ds^2}=\frac{d^2z}{ds^2}=0 \qquad (7a)$$

where

$$\Gamma^t{}_{tt} = \frac{d}{dT}[\ln\{\exp(T/C) + \exp(-T/C)\}]. \qquad (7b)$$

Eq. (7) implies that there is no acceleration to cause a local transformation. Then, it follows eq. (7) that

$$\frac{dT}{ds} = k\{\exp(T/C) + \exp(-T/C)\}^{-1} \qquad \text{and} \qquad \frac{dx^\mu}{ds} = \text{constant}, \ x^\mu \ (= x, y, z) \qquad (8)$$

for some constant k. Now, consider the observer P in the state

$$dx/dT = dy/dT = dz/dT = 0; \ \text{and thus} \quad dx/ds = dy/ds = dz/ds = 0 . \qquad (9)$$

This means P would have the same frame of reference whether "free falling" or not.

Consider the local Minkowski space of P at (x_0, y_0, z_0, T_0)

$$ds^2 = c^2dT'^2 - dx'^2 - dy'^2 - dz'^2 , \qquad (10)$$

The geodesic of P is (x_0, y_0, z_0, T) and the local coordinate transformation to the local Minkowski space is

$$dx' = dx, \ dy' = dy, \ dz' = dz, \text{ and } \ dT' = ch(T/C)dT . \qquad (11)$$

Again, (11) is invalid in physics because there is no physical cause, neither velocity nor acceleration.

Moreover, $ds^2 = 0$ would give [1] the light speed to be ±c ch (T/C). Such an observer P would absurdly have two different light speeds (see § 5) from the same frame of reference. Moreover, metric (6) is obtained from the flat metric

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2, \qquad \text{with} \qquad t = C\ sh(T/C), \qquad (12)$$

a diffeomorphism. Although the geodesic equation (10) is mathematically covariant, the time coordinate is not. It is the wrong physical interpretation, which is based on a non-physical coordinate system of the space that leads to the difficulties discussed above[5].

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[5] There are theorists who deny any physical meaning of space coordinates [6,7,12] because they cannot tell the difference between mathematics and physics. They should read carefully Einstein's 1921 article,

This example illustrates that Einstein [1,2] is “obliged to define time in such a way that the rate of a clock depends upon where the clock may be” because Einstein's equivalence principle would not be satisfied otherwise.

In general relativity, Einstein’s measuring instruments are at rest but in *a free fall state*.[1] Based on such measurements, Einstein believed, “In the general theory of relativity, space and time cannot be defined in such a way that differences of the spatial coordinates can be directly measured by the unit measuring-rod, or differences in the time coordinates by a standard clock”. An implicit assumption is that the measuring instruments are unchangeable as in Riemannian geometry.

However, if the measuring rod is attached to the frame of reference, since the measuring rod and the coordinate being measured are under the same influence of gravity, the Euclidean-like structure emerges as a result. This clarifies the seemingly inconsistency between a curved space and the Euclidean-like structure and that the meaning of spatial coordinates is actually independent of gravity. It is on this basis that Einstein reached his conclusion of space contraction. Then, the time coordinate is determined by orthogonality such that the time rate is the same as a local clock. It will be shown that these are supported by experiments.

Note that the Euclidean-like structure is a notion in physics due to Einstein’s equivalence principle, and exists only in a physical space although it may appear that such a structure exists in a mathematical Riemannian space. Since the physical meanings of the coordinates are clarified, Einstein’s equivalence principle, which is defined in terms of the local equivalence between gravity and acceleration with respect to a frame of reference, is now clear if it was not [7] (see also the Appendix).

To illustrate these, let us consider a spherical mass distribution with total mass M, and the Schwarzschild solution is

$$ds^2 = (1 - 2MG/\rho)dt^2 - (1 - 2MG/\rho)^{-1}d\rho^2 - \rho^2 d\theta^2 - \rho^2 \sin^2\theta d\varphi^2, \qquad (13a)$$

where

$$x = \rho \sin\theta \cos\varphi, \; y = \rho \sin\theta \sin\varphi, \; \text{and} \; z = \rho \cos\theta , \qquad (13b)$$

(13a) is a function of ρ (= $[x^2 + y^2 + z^2]^{1/2}$), and (x, y, z) is related to the Euclidean subspace when M = 0. Since the Euclidean-like structure (13b) of a frame of reference is independent of gravity, it is justified to choose a frame a priori to the solution.[1,2]

A major problem in general relativity was that Einstein was unable to define the space coordinates in terms of physics.[1,2,27] Although Einstein discovered the Euclidean-like structure, he failed to see the related physics. Due to such ambiguity, Einstein’s equivalence principle is not yet precisely defined in physics. Consequently Whitehead [26] rejected general relativity as a physical theory, and Einstein’s followers have to replace Einstein’s equivalence principle with Pauli’s version.[5]

“Geometry and Experience” [41]. A related paper on the physical meaning of space-time coordinates and its further experimental verification has been published in the Chinese Journal of Physics.[42]

4. Einstein's Demonstration of Einstein's Equivalence Principle

Having shown that Einstein's equivalence principle may not be satisfied in a Lorentz manifold, an application of the (Einstein's) equivalence principle for a physical situation should be illustrated. To this end, Einstein's own application would serve as good examples. Also, remarks are added to along his calculations to clarify some points discussed in the previous section.

Einstein [1] illustrated the application of his equivalence principle to a solution of his equation and obtained the three famous tests for static gravity: the perihelion of Mercury, gravitational red shifts, and the light bending around the sun. The importance of physical validity of the geodesic was demonstrated by the fact that Einstein's own confidence on general relativity was based on obtaining the numerical value for Mercury perihelion by modifying his field equation.

First, he modeled the sun as a static spherical symmetric star, and considered a coordinate system S with the sun attached to the spatial origin. Then, Einstein derived $g_{\mu\nu}(x, y, z, t)$, with his notion of weak gravity, the linear equation[6],

$$\frac{\partial^2 \gamma_{\mu\nu}}{\partial x_\alpha^2} = 2K(T_{\mu\nu} - g_{\mu\nu}T), \text{ where } \gamma_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}, \ T = T_{\alpha\beta}g^{\alpha\beta} \quad . \qquad (14a)$$

$\eta_{\mu\nu}$ is the flat metric, $T(m)_{\mu\nu}$ is the energy tensor for massive matter, and K is the coupling constant. The coordinates are chosen independent of a solution of $g_{\mu\nu}$ and this is justified since the Euclidean-like structure is independent of gravity. Then, from eq. (14a), he obtained, to a sufficiently close approximation, the static metric for coordinate system S

$$ds^2 = (1 - \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0})dt^2 - (1 + \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0})(dx^2 + dy^2 + dz^2) \qquad (14b)$$

where

$$r_0 = [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2} \ ; \qquad (14c)$$

is the distance of the Euclidean-like structure, which is independent of (14b), and $\sigma(x_0,y_0,z_0)$, is the mass density.

To show that the correspondence principle is satisfied, Einstein derived the Poisson equation of Newton's theory with eq. (14a) and Newton's equation of motion with the geodesic equation. Also, he justified the validity of his geodesic equation by calculating

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[6] The Maxwell-Newton Approximation (14) has been derived, independent of Einstein equation and the harmonic condition, with only three assumptions: 1) Einstein's equivalence principle; 2) Special relativity; and 3) Newtonian theory as a first order approximation for gtt [20]. (Such an independency is necessary because the linear equation for weak gravity and the 1915 Einstein equation are incompatible for a dynamic case.) It has been predicted [30] that the Stanford Gravity Probe-B gyroscopes on the precessions would further confirm this Approximation. It is expected also that the experiments on local light speeds [33] will directly support the Maxwell-Newton approximation, but reject the Schwarzschild solution.

the perihelion of Mercury. Note that eq. (14a) can be justified, independent of the 1915 equation, directly with physical requirements, which include Einstein's equivalence principle and Newton's gravity as a first order approximation.[20]

According to Einstein's equivalence principle, gravitational static acceleration should exist. The principle of causality[4)] requires that gravity should have causes. From the geodesic eq. (5), one has $d^2x^\mu/ds^2 \neq 0$ for x^μ (= x, y, z) since $\partial g_{tt}/\partial x^\mu \neq 0$ due to non-zero mass distribution. Thus, the equivalence principle would be applicable

Consider an observer P, in a free falling state, at (x, y, z, t) with

$$dx/ds = dy/ds = dz/ds = 0 \quad . \tag{15}$$

Assuming that the equivalence principle is applicable, metric (14) and state (15) imply the time dt and dT are related by

$$c^2\left(1 - \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0}\right)dt^2 = ds^2 = c^2 dT^2 \tag{16}$$

since the local Minkowski coordinate system is attached to the observer P (i.e., dX = dY = dZ = 0 in eq. [1]). Also, since the space coordinates are orthogonal to dt, at (x, y, z, t), for the same ds^2, eq. (16) implies [3]

$$(dx^2 + dy^2 + dz^2)\left(1 + \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0}\right) = dl^2 = (dX^2 + dY^2 + dZ^2) \ . \tag{17}$$

Equation (17) leads Einstein to conclude space contraction[7]. However, this declaration requires that dx, dy, and dz are defined independent of the equivalence principle. Now, the Euclidean-like structure clarifies the physical meaning of these coordinates. Since the acceleration $d^2x/ds^2 \neq 0$, gravitational red shifts are expected. From eq. (16), one obtains

$$\nu(x, y, z) = \nu_0\left[1 - \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0}\right]^{-1/2} \approx \nu_0\left[1 - \frac{K}{8\pi}\int dV_0 \frac{\sigma}{r_0}\right] = \nu_0[1 + \Phi c^{-2}] \ , \tag{18a}$$

where

$$\Phi(x, y, z) = -\frac{K}{8\pi}\int dV_0 \frac{\sigma}{r_0} = \frac{M}{r} \quad \text{where} \quad r = [x^2 + y^2 + z^2]^{1/2}, \quad \text{and} \quad M = \int dV_0 \sigma \tag{18b}$$

is the total mass, $\nu(x, y, z)$ and ν_0 are respectively the frequencies in the sun surface and the flat space. Thus, since Φ is the Newtonian potential, the gravitational red shift

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[7] This situation is different from the case when one chooses a non-inertial (Galilei) frame to describe a free particle. In such a case, independent of regarding the appeared “fictitious forces” as real forces or not, no inconsistency in physics would result. However, when the equivalence principle is applied to a non-physical space, one obtains unphysical results. This is different from an integrated effect such a light deflection, which can be obtained from an unphysical metric.

formula agree with the one Einstein derived in 1911 from his principle.[21] This is another confirmation that the equivalence principle would be satisfied.

From eqs. (16) and (17), Einstein was confident in deriving the light speed at (x, y, z),

$$\frac{[dx^2 + dy^2 + dz^2]^{1/2}}{dt} = c(1 - \frac{K}{4\pi}\int dV_0 \frac{\sigma}{r_0}) \quad . \tag{19}$$

to the first order approximation[8]. The light speed (19), for an observer P_1 attached to the system S at (x, y, z), is smaller than c. Observer P_1 shares the same frame of reference with the sun, and the velocity of light is clearly frame-dependent, but restricted. The observer P is in a free falling frame of reference and thus would not experience the gravitational force as P_1.

Einstein wrote [1], "We can therefore draw the conclusion from this, that a ray of light passing near a large mass is deflected." Thus, the deflection of light supports that the Euclidean-like structure has an objective existence in nature. Currently, theorists derived this deflection directly without using the light speeds, which depends on the coordinates. However, such a derivation still depends the space coordinates, which are necessarily used to define the deflection angle. Moreover, there are theorists such as Liu Liao [12] who defined the light speed in terms of the local distance and coordinate time, i.e., dl/dt = c $(g_{tt})^{1/2}$, although this would lead to an incorrect deflection angle. Yu [9] invalidly claimed that all measurable quantities must be scalars.

Although eq. (16) is necessary for (18), one could have obtained eq. (19) directly from

$$ds^2 = 0 \quad . \tag{20}$$

The reason of doing eqs. (16) and (17) is to see if the equivalence principle would be satisfied for (18)[9]. Moreover, (16) and (17) make clear that, *in Einstein's equivalence principle, the local spaces of a geodesic are to be Minkowski.*

From eq. (19), a light ray passing at a distance Δ from the origin, will be deflected, in all, by an amount

$$\alpha = \frac{KM}{2\Delta\pi} \tag{21}$$

towards the sun. If Δ equals to the radius of the sun, α amounts to 1.7". Although eq. (21) can be derived with a null geodesic [3-5], the frame of reference attached to the sun is needed to define the deflection angle.

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[8] An implicit assumption of Einstein's calculation is that the gravity due to a light ray is negligible. However, this experimentally valid assumption was not supported by the current theory of general relativity [34]. This shows that Einstein's field equation is not as logically complete as Einstein [31] and most theorists believed.

[9] However, this criterion is actually inadequate since there are more than one metrics that satisfy it. Moreover, the second order of gravitational red shifts is not gauge invariant as Einstein claimed [1].

Moreover, since a light ray can be scattered by the gravitational field, it is possible that the scattered photons would loss energy as in any scattering theory. Since gravity is very weak, such a small energy lost would be observable only as an accumulated effect after traveling over a long distance in terms of light years, and on the average would increase proportional to the distance. In other words, gravitational scattering should be a cause of the observed redshifts from distant stars. However, such a dynamical situation cannot be addressed quantitatively yet because of the limitation of Einstein equation.[29,30]

The resulting displacement of the perihelion of planetary orbits in the direction of the planet's orbital motion is

$$\frac{24\pi^3 a^2}{c^2 T^2 \left(1 - e^2\right)} \tag{22}$$

where *a* is the semi-major axis (in terms of the Euclidean-like structure) of the planetary orbit in centimeters, *T* is the period of revolution in seconds, *e* is the numerical eccentricity, and *c* is the speed of the light in vacuo. The semi-major axis is evaluated with Euclidean geometry eq. (19c). Here the second order approximation calculation will not be reproduced. Einstein [1] wrote, "This expression furnishes the explanation of the motion of the perihelion of the planet Mercury, which has been known for a hundred years (since Leverrier), and for which theoretical astronomy has hitherto been unable satisfactorily to account." Note that in the above derivations, in effect, Einstein had stated that *the Euclidean-like structure is measurable.*

5. Experimental Restrictions on Space-Time Coordinates

It will be shown that, due to the observed gravitational redshifts, there is an important theoretical implication on covariance because valid space-time coordinate systems are restricted by Einstein's equivalence principle. Due to the physical requirement that eq. (18a) must be compatible with eq. (3), a diffeomorphic transformation of the system S(x, y, z, t) is restricted.

To show this, let us present eq. (18a) alternatively as

$$\nu_1 \,[g_{tt}(r_1,t_1)]^{-1/2} = \nu_2 \,[g_{tt}(r_2,t_2)]^{-1/2} , \tag{23a}$$

where ν_i is the frequency at (r_i,t_i). If diffeomorphic systems were always equivalent, consider the diffeomorphism (12), t = C sh(T/C). Then, for the new coordinate system (x, y, z, T), one obtains

$$g_{TT}(r,T) = g_{tt}(r,t)\, ch^2(T/C) = g_{tt}(r)\, ch^2(T/C) \,; \tag{23b}$$

and

$$\nu_1[g_{TT}(r_1,T_1)]^{-1/2} = \nu_1[g_{tt}(r_1) ch^2(T_1/C)]^{-1/2} = \nu_2\, [g_{tt}(r_2) ch^2(T_2/C)]^{-1/2} \tag{23c}$$

would follow. Obviously, eq. (23c) does not agree with formula (23a) nor (18), and is incompatible with eq. (3) which was derived with Einstein's equivalence principle. It should be noted that coordinate systems S(x, y, z, t) and S'(x, y, z, T) shares the same frame of reference. Thus, once the spatial coordinates are chosen, the time coordinate is also determined.

In short, the bending of light supports that the frame of reference has the Euclidean-like structure as required. The gravitational redshift formula (18) supports Einstein's notion of the local clock rate, and shows that a space-time coordinate system is necessarily restricted by Einstein's equivalence principle.

6. Conclusions and Discussions

Einstein's equivalence principle requires that a free falling observer must move along a geodesic and result in a co-moving local Minkowski space. It is essential to understand, just as in special relativity, the local space is not arbitrary, and thus the physics related to such a local coordinate transformation must be investigated. As Einstein asserted, the equivalence principle is necessary to ensure that [2] "special theory of relativity applies to the case of the absence of a gravitational field." It follows that the first experiment that supports Einstein's equivalence principle is actually the Michelson-Morley experiment.

Mathematically, a non-constant Lorentz metric may not imply a "gravitational acceleration" to a static particle. Thus, Einstein's equivalence principle is not equivalent to the existence of local Minkowski spaces. Moreover, the correspondence principle requires that for weak gravity the related metric must be approximately Minkowski. This continuity is provided by Einstein's equivalence principle, but validity of an arbitrary Gaussian coordinate system would mean a disruption of this continuity.

Einstein's equivalence principle is proposed for the reality. Thus, to apply this principle in a manifold, one should try to justify that this manifold is a physical space as Einstein did. Since physical requirements have not been described precisely, it would be difficult to present general relativity in a manner as mathematics is presented. Einstein [2] stated in 1916 the following:

> *"It is not my purpose in this discussion to represents the general theory of relativity as a system that is as simple and logical as possible, and with the minimum number of axioms; but my main object is to develop this theory in such a way that the reader will feel that the path we have entered upon is psychologically the natural one, and that the underlying assumptions will seem to have the highest possible degree of security."*

Moreover, it is difficult to describe the physical process (due to a free falling), which locally transforms the metric to a local Minkowski space automatically although he infers the correct result. These may explain that Eddington [28,31] cannot find a third person, who understands general relativity, and the situation has not been improved much since. In the literature, there are only a few authors [32,33] who discussed space contractions -- an implication of Einstein's equivalence principle.

Coordinates of a physical space must have physical meanings because the validity of physical interpretations depends on such a system of coordinates. Although a tensor equation in physics is covariant, physics must be based on understanding the components of a tensor. It follows that a valid coordinate system in physics cannot be just a Gaussian system in mathematics. In addition, Einstein's equivalence principle is not simply a tensor equation.

The Michelson-Morley experiment is a proof that a constant metric must be the restricted. Another proof for the restriction of Einstein's equivalence principle on the validity of a coordinate system for physics is the formula for gravitational red shifts. The validity of the light bending, which is produced by the coordinate light speeds, supports the theoretical physical meaning of space coordinates in terms of the Euclidean-like structure.

The formula for gravitational redshifts provides the experimental evidence that once a frame of reference is chosen, the time coordinate is decided. The theoretical usage of the physical meaning of coordinates started from considering the symmetry for the circle in a rotating frame [1] because the principle of causality was applied. Nevertheless, a fundamental problem among relativists is that they cannot really tell the difference between mathematical and physical coordinate systems.

In conclusion, the equivalence principle is a theoretical framework and it restricts the covariance. The mathematical theorems show only that Riemannian geometry is compatible with general relativity. The physical meaning of space-time coordinates is crucial. Due to inadequate understanding of coordinates, Einstein's equivalence principle was practically dismissed [6] and therefore physical requirements on a Lorentz manifold cannot be adequately considered. This analysis shows that the ability to identify physical requirements is necessary for the understanding of general relativity.[20,30,34]

Appendix: Physical Conceptions and Misconceptions Related to Einstein's Equivalence Principle

Einstein [1] remarked, "As in special theory of relativity, we have to discriminate between time-like and space-like line elements in the four-dimensional continuum; owing to the change of sign introduced, time-like line elements have a real, space-like line elements an imaginary ds. The time-like ds can be measured directly by a suitably chosen clock." Thus, a space-coordinate and the time-coordinate are not exchangeable as Hawking [35] claimed since they have distinct characteristics. (However, this does not prevent Hawking from asserting in the same book that time has an arrow.)

Many theorists believe the validity of any Gaussian system as a space-time coordinate system in physics [6]. Since they believed also that space-time coordinates have no physical meaning, they regarded Einstein's principles as heuristic. This leads incorrectly to that Einstein's equivalence principle must be replaced by the condition for the mathematical existence of a local Minkowski space. On the other hand, in Einstein's theory [1,2], for a space-time coordinate system, "the rate of a clock depends upon where the clock may be." Thus, only some of the Gaussian systems can be used as a space-time coordinate system.

Although Einstein had argued for an unrestricted covariance in 1916, perhaps due to the criticisms of Eddington [24], Einstein [1] dropped those arguments in his book, "The Meaning of Relativity". He also made clear that general covariance is beyond the principle of general relativity, which states only the equivalence of all frames of reference in different kinds of motion. Note that there are physics laws such as Einstein's equivalence principle, which is not just a tensor equation. Nevertheless, this principle is invariant with respect to any physical space, which has a frame of reference in arbitrary motion.

Moreover, if coordinates had no objective meaning, a tensor component would have no physical meaning unless it can be derived in coordinate free notations, i.e., scalars. However, a tensor component and a scalar can be numerically equal only for some special coordinate systems. If such an equality is meaningful in physics, the related special coordinate system must have physical meaning. In other words, the usage of a physically meaningful coordinate system is necessary because simply non-scalars are involved in physics. Thus, the viewpoint that coordinates have no meaning in physics is logically self-defeating.

Now, let us consider the question of representing a light frequency ν. A frequency is not a scalar, and due to the Doppler effect, the frequencies are different in coordinate systems with relative velocity. In a flat metric, the light frequency ν is the time component of propagation vector, and can be presented as a scalar.[6,36] However, time dilation based on Einstein's equivalence principle and Einstein's slower clock interpretation are crucial to allow such a scalar be considered as the frequency under gravity. Thus, the so-called coordinate-free derivation implicitly requires a valid space-time coordinate system, which satisfies Einstein's equivalence principle. Currently, the light deflection is derived directly without calculating the light speed as Einstein did. However, this derivation also requires a physical coordinate system that the deflection angle can be defined.

Currently, Pauli's version [5] "is now commonly but mistakenly regarded as Einstein's version of the principle [37]" in spite of the fact that Einstein strongly objected Pauli's version as a misinterpretation.[37] Nevertheless, Pauli's version though inadequate in physics, is popular because physical requirements beyond metric signature are ignored.

For the convenience of discussion, let us state first Pauli's "equivalence principle" as the following:

> *"For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system K_0 (X_1, X_2, X_3, X_4) in which gravitation has no influence either in the motion of particles or any physical process."*

It is the claim of Pauli that gravity can be transformed away leads to the misleading notion of equivalence between gravity and acceleration although Einstein objected. Nevertheless, many theorists regarded it as Einstein's [4]. Moreover, Bergmann [38] further affirmed this misconception with his notion of "Einstein elevator".

Different from Pauli's version, Einstein requires additionally: i) "the special theory of relativity applies to the case of the absence of a gravitational field [2, p.115]", ii) "he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be [2, p.116]." and iii) a local Minkowski space is obtained by choosing the acceleration due to gravity. Einstein [2, p.118] wrote, "... we must choose the acceleration of the infinitely small ("local") system of coordinates so that no gravitational field occurs; this is possible for an infinitely small region. Point i) makes clear that the Minkowski space is the only valid constant metric when gravity is absent. Point ii) makes clear that a physical space must have a physically valid space-time coordinate system. Point iii) implies that acceleration to a static particle must exist.

Einstein regards that the resulting local space being Minkowski is a physical must. Thus, the cause and consequences of such local coordinate transformation must be investigated in terms of physics.[1,2] Moreover, there are Lorentz manifolds that are not diffeomorphic to a physical space.[19,20] Therefore, it is beyond doubt that Pauli's version is inadequate in physics. Pauli also believed, in disagreement with Einstein [39,40], "a gravitational mass $m = E/c^2$ has to be ascribed to an energy E in all cases".

Moreover, Fock [25] had mistaken that a frame of reference is related to a Euclidean subspace instead of just the Euclidean-like structure. (Note that the covariant and the contravariant forms of tensors are still based on the space-time metric.) The necessary existence of the Euclidean-like structure shows that Einstein's physical space is a special kind of Riemannian space that is new even in mathematics. Thus, according to Einstein's equivalence principle, one cannot regard a Riemannian space as a physical space without examining the specific frame of reference and the time-coordinate used. On the other hand, the existence of a mathematical Euclidean-like structure in metric (6) need not mean the manifold is a physical space. A problem of Einstein's theory is that the meaning of coordinates was not clearly defined [25,26], and this makes his principles essential inapplicable.

To illustrate the notion of physical Euclidean-like structure further, let us consider a Lorentz Manifold with the metric,

$$ds^2 = c^2dt^2 - ch^2(X/C)dX^2 - dy^2 - dz^2 \qquad \text{(A1)}$$

where C is a constant. Consider an observer P resting at the frame (X, y, z). P would rest forever at the same frame of reference. However, if the equivalence principle were valid, such an observer P would have two different light speeds in the X-direction from the same frame of reference. Thus, metric (A1) does not have a physically valid Euclidean-like structure.

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