Another View of Newtonian Gravity

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Introduction: We are still unsure if gravity is the effect of masses pulling on each other or whether some remote force carrying media such as a galactic gravitational field is compressing the masses towards each other. We will show that all Newtonian gravitation theory applies to a good approximation and that the curved space of Einstein’s theory becomes easier to understand, and also show that some of the anomalies associated with determining a precise value of Newton’s Gravitational Constant \( G \), can be explained.
ANOTHER VIEW OF NEWTONIAN GRAVITY by William F Tottle (Part 1)

Introduction
We are still unsure if gravity is the effect of masses pulling on each other or whether some remote force carrying media such as a galactic gravitational field is compressing the masses towards each other. R1 We will show that all Newtonian gravitation theory applies to a good approximation and that the curved space of Einstein’s theory becomes easier to understand, and also show that some of the anomalies associated with determining a precise value of Newton’s Gravitational Constant $G$, R4, can be explained.

1. A Plot of the Earth’s and Moon’s Acceleration $g_e$ and $g_m$ Against Linear Distance $d$

1.1 Gravitational Acceleration Moments about $P_1$ see Figure 1
The gravitational acceleration on the Earth’s surface due to the Earth’s mass is:

$$g_e = -\frac{GM_e}{R_e^2}, \quad (R2)$$  \hspace{1cm} 1.1.1

and the g-acceleration on the Moon’s surface due to the Moon’s mass is:

$$g_m = \frac{GM_m}{R_m^2}, \quad 1.1.2$$

so that at $P_1$ the g-accelerations due to the Earth’s and the Moon’s mass respectively are:

$$g_{(e-P_1)} = -\frac{GM_e}{d_{(e-P_1)}^2}, \quad 1.1.3$$

and $$g_{(m-P_1)} = \frac{GM_m}{d_{(m-P_1)}^2}, \quad 1.1.4$$

which sum to zero, so the moments of gravitational acceleration are:

$$M_e^{-2} \cdot d_{(e-P_1)} = M_m^{-2} \cdot d_{(m-P_1)}, \quad 1.1.5$$

1.2 Gravitational Potential Moments about $P_1'$. Note: $P_1' \neq P_1$.

The gravitational potential $U_e$ at the surface of Earth is given by:

$$U_e = -\frac{GM_e}{R_e}, \quad 1.2.1$$

and

$$U_m = \frac{GM_m}{R_m}, \quad 1.2.2$$

which also sum to zero at $P_1'$, so the g-potential moments about $P_1'$ are: $M_e^{-1} \cdot d_{(e-P_1)'} = M_m^{-1} \cdot d_{(m-P_1)'}$.  

As the gravitational potentials are equal and opposite at $P_1'$ then so the Moon’s and Earth’s forces are equal and opposite, in fact there will be a line of zero force perpendicular to the line joining the two bodies centres of which $P_1'$ is just one point. This is not shown in figure 1, but referring to equation 1.4.2, $d_{(e-P_1)'} = 3.7977661 \times 10^8$ metres which in the Figure would be slightly to the right of $P_1$. At every point on the line of equilibrium of forces $f_e + f_m = 0$ and the figure shows an idealised situation which assumes there is no influence from other astral bodies.

1.3 The units of the areas enclosed by the in Figure 1 are in $m^2.s^{-2}$ which are the units of force in Newtons. So the areas enclosed represent g-force fields in the plane containing the centres of the earth and the Moon.

For the Earth outside: $$\int_{R_e}^\infty -\left(\frac{GM_e}{d^2}\right) \, d = \left[\frac{GM_e}{(2d)}\right]_{R_e}^\infty = -\frac{GM_e}{(2R_e)}$$ for the plane.

Inside: $-9.8R_e/2 = -4.9R_e$, assuming uniform density.

So the total g-force field is: $-\left[\frac{GM_e}{(2R_e)} + 4.9R_e\right]$.

and for the cone of force: $-2\pi\left[\frac{GM_e}{(2R_e)} + 4.9R_e\right]$ Newtons  \hspace{1cm} 1.3.1

Calculation of the force fields for inside and outside for the Moon, Sun and Earth, where the external distances are linear as in the Newtonian theory suggest the following. This would be the case for a single massive body in the universe.

**Magnitude of Internal Force Field = Magnitude of external Force Field**
We take the Moon as an example:

External: $$GM_m/(2R_m) = 6.67259 \times 10^{11} \times 7.349 \times 10^{22} / (2 \times 1.738 \times 10^9)$$

$$= 1.41 \times 10^6 \text{ N.}$$

Internal: $$g_mR_m/2 = 1.623 \times 1.738 \times 10^6 / 2$$

$$= 1.41 \times 10^6 \text{ N.}$$

so, $$GM_m/(2R_m) = g_mR_m / 2,$$

from which, $$g_m = GM_m / R_m^2, \quad 1.3.2$$

and the assumption for the Moon is confirmed and applies to all massive bodies of spherical shape.
1.4 Point P₁ lies on the line joining the centres of the Earth and Moon, therefore the radius vector of the Moon in its orbit \( R_{(e-m)} \) equals the sum of \( d_{(m-P₁)} \) and \( d_{(e-P₁)} \), so \( d_{(m-P₁)} = R_{(e-m)} - d_{(e-P₁)} \). Now \( g_m = GM_m / d_{(m-P₁)}^2 \) and \( g_e = -GM_e / d_{(e-P₁)}^2 \), are the respective gravitational potentials at \( P₁ \) and these sum to zero at the point of equilibrium. Hence,

\[
GM_e / d_{(e-P₁)}^2 = GM_m / (R_{(e-m)} - d_{(e-P₁)})^2,
\]

which on simplification leads to the solution of a quadratic in \( d_{(e-P₁)} \). The solution of which is:

\[
d_{(e-P₁)} = 3.4619877 \times 10^8 \text{ m},
\]

and therefore,

\[
d_{(m-P₁)} = 3.820223 \times 10^7 \text{ m}.
\]

From equation 1.2.3,

\[
d_{(e-P₁)} = d_{(m-P₁)} M_e / M_m,
\]

so,

\[
= R_{(e-m)} M_e / (M_m + M_e)
\]

and,

\[
d_{(m-P₁)} = 4.6243928 \times 10^6 \text{ m}.
\]

1.5 The Major Line of Acceleration Due to Gravity Between the Surfaces of Earth and the Moon

The **major line of acceleration** is defined as the shortest line of gravitational potential \( U \) attributable to some function of two or more masses.

Figure 2 (not to scale) shows the Earth and Moon separated by the radius vector \( R_{(e-m)} \). The total lengths of the curved paths are made up of two arc lengths between the surfaces of the two bodies, plus the radius of Earth \( R_e \) and the radius of the Moon \( R_m \). The centres of both arcs are on the line of equilibrium of forces which is perpendicular to the centre line and passes through \( P₁' \). The circles of equipotential whose centres lie on the centre line, cut the lines of force at right-angles. The Figure pictorially shows the principle on which the curvature theory is based.

A *gravitational field* is defined as significant in a region if the gravitational forces exerted on masses in that region are of sufficient strength to make observational measurements of its influence possible.

The *direction of a g-field* at a point is defined as the direction in which a unit mass (1kg) would move under the influence of the field, if placed at that point.

The *gravitational field intensity* or *gravitation field strength* \( (g) \) at a point is defined as the force exerted by the field on a unit mass placed at that point. Units: N.M⁻¹ or m.s⁻²

It follows from the definition of gravitational field intensity that the force \( f \), exerted on a mass \( M \), at a point where the field intensity is \( g \), at the site of \( M' \) is given by:

\[
f = Mg.
\]

1.6 The Gravitational Force Acting Between Two Massive Bodies

For convenience we take the two bodies as being Earth and the Moon, so the radii are given as \( R_e \) and \( R_m \), respectively. The assumed spherical masses are considered to be isolated from all other gravitational influence. We can find an expression for the gravitational force acting between the two bodies by considering equation 1.1.3,

\[
g_{(e-P₁)} = -GM_e / d_{(e-P₁)}^2
\]

From equations 1.5.1 & 1.6.1 the force acting on a unit mass at \( P₁' \) is given by:

\[
f_{(e-P₁)} = -GM_e 1\text{kg} / d_{(e-P₁)}^2
\]

And if we place \( P₁' \) at the centre of the Moon the \( d_{(e-P₁)} \) has changed to \( R_{(e-m)} \) the Moon’s radius vector and the 1kg has changed to \( M_m \) the Moon’s mass, so the equation becomes:

\[
f_{(e-m)} = -GM_e M_m / R_{(e-m)}^2
\]

where \( R_{(e-m)} \) is the linear distance between the centres of Earth and the Moon. Equation 1.6.3 is modified when the gravitational influence of the other celestial bodies particularly the Sun is considered, which tends to bend the lines of gravitational force into arcs, so the distance increases slightly to \( R_{(e-m)}' \) resulting in a drop in force. So the general equation of force acting between two massive bodies \( M_1 \) and \( M_2 \) is:

\[
f_{(1-2)} = -GM_1 M_2 / R_{(1-2)}^2
\]

the negative sign indicating that the force acts towards the centre of \( M_1 \).

1.7 Field Intensity Due to a Point Mass

The force \( f \) acting toward the centre of a test mass \( M' \), due to a point mass \( m \) of 1kg in free space is given by:

\[
f_{M'} = GM' m / R_{(M'-m)}^2
\]
1.8 Gravitational Flux

The flux lines in a gravitational field will in general be changing direction and density from place to place. They also form a three dimensional picture. Figure 3 shows a curved surface \( S \) near a mass \( M \). The lines of gravitational force due to \( M \) will cross the surface in different directions and two such lines are shown. They act downward on the surface at points \( P_1 \) and \( P_2 \), which are small areas \( dS_1 \) and \( dS_2 \) respectively, of \( S \). If \( dS_1 \) and \( dS_2 \) are small enough they can be considered flat. \( P_1 N_1 \) is a normal to the surface at \( P_1 \) and \( P_2 N_2 \) is the normal at \( P_2 \). There will be an infinite number of small areas like \( dS_1 \) and \( dS_2 \) which go to make up \( S \). Hence, the normal component of gravitational flux density vector \( \mathbf{D} \) at \( P_1 \) is \( \mathbf{D}_1 \cos \theta \) and \( \mathbf{D}_2 \cos \theta_2 \) for \( P_2 \). All of the surfaces like \( dS_1 \) which go to make up \( S \) each have a component of g-flux density at right-angles at a point such as \( P_1 \) on the surface. \( \mathbf{D}_i \cos \theta_i dS_i \) is called, `the gravitational flux density across the surface \( dS_i \)'. (flux density times area equals flux). Similarly, \( \mathbf{D}_2 \cos \theta_2 dS_2 \) is the g-flux across the surface \( dS_2 \). The symbol for g-flux is \( \psi \) (psi). For every area like \( dS_1 \) there will be a corresponding \( D \cos \theta dS \) and the flux across the whole surface \( S \) is the sum of all of these. This is written as a surface integral:

\[
\int_S D \cos \theta dS
\]

For the sphere the sum of all the areas like \( dS_1 \) which make up the surface \( S \), is simply the surface area of the sphere \( 4\pi r^2 \).

1.9 Gauss’ Theorem for Gravitation

One of the most important theorems in gravitation and being able to apply it is the key to understanding and development. The gravitational field strength measured in units of \( \text{kg.m.s}^{-2} \) at a point in a gravitational field is defined as the force per unit mass acting on a mass at that point. A closed surface is defined as one without edges eg a sphere.

The closed surface shown in Figure 4 is made up of an infinite number of small areas like \( dS_1 \). The direction of the \( \mathbf{g} \)-vector and hence the \( \mathbf{D} \)-vector at \( P_1 \), a point on \( dS_1 \), due to the mass \( M \) at \( O \) is at an angle \( \beta \) to the normal to the surface \( dS_1 \) at \( P_1 \), where the g-field strength is given by:

\[
g_1 = -\frac{GM}{d_1^2},
\]

which for Earth is:

\[
g_e = -\frac{GM_e}{R_e^2},
\]

and the g-flux density,

\[
D_1 = -\frac{M}{d_1^2},
\]

which for Earth is:

\[
D_e = -\frac{M_e}{R_e^2} = g_e / G \text{ kg.m}^2.
\]

Therefore the g-flux density \( d\psi \) across surface \( dS \) is:

\[
d\psi = D \cos \beta dS
\]

\[
d\psi = -M \cos \beta_1 dS_1 / d_1^2.
\]

We now concentrate on \( dS_1 \). In fact g-flux density will have a large changing horizontal component and a small changing vertical component because the line of gravitational potential is not normal but an arc. In Figure 5 a sphere of radius 1 metre is drawn with centre \( O \) so that it encloses mass \( M_1 \). If \( O \) is joined to every point on the perimeter of \( dS_1 \) by means of straight lines, a cone is formed with its apex at \( O \).

The area \( dS_1 \cos \beta_1 \) is a section of the cone through \( P_3 \) and perpendicular to \( OP_1 \), where

\[
s P_3P_1 = dS_1 \sin \beta_1 dS_2
\]
the sphere \( S_2 \) by the cone lines. We will show that the g-flux crossing any closed surface is the same as that crossing a sphere of unit radius enclosing the same mass and is in fact equal to \( 4\pi \) times the mass \( M_1 \) enclosed. By similar triangles:

Area \[
\frac{dS_2}{(\text{Area} \_ dS_1 \cos \beta_1)} = \left[\frac{(OP2)}{(OP1)}\right]^2 = d_1^{-2}, \text{ where } d_1, \phi, dS_1 \sin \beta_1
\]

so,

\[
dS_2 = dS_1 \cos \beta_1 / d_1^2, \tag{1.9.8}
\]

and,

\[
dS_1 = dS_2 d_1^2 / \cos \beta_1. \tag{1.9.9}
\]

Using equations 1.9.6 and 1.9.9 we have the g-flux across the surface \( S_1 \): \[
d\psi = -M \cos \beta_1 dS_2 d_1^2 / (d_1^2 \cos \beta_1),
\]

Hence

\[
d\psi = -MdS_2. \tag{1.9.10}
\]

The total g-flux across the whole surface \( S_1 \) is therefore the sum of all the \( d\psi \) terms which means adding all the \(-M.dS\) terms over the surface \( S_2 \).

\[
\psi = -\int_{S_2} MdS, \text{ where } M \text{ is a constant,}
\]

so,

\[
\psi = -4\pi M \text{ kg } (4\pi.1^2 \text{ is the surface area of a sphere of unit radius}) \text{ hence,} \tag{1.9.11}
\]

If there is more than one mass enclosed by the surface \( S_1 \), we use the Principle of Superposition to show that the g-flux crossing the surface is proportional to the sum of the masses enclosed.

\[
\psi = -4\pi \sum_{n=1}^{\infty} M_n, \text{ } n \in \{1,2,3,...\} \tag{1.9.12}
\]

This is Gauss’ theorem and is stated as follows:-

‘The net gravitational flux (entering) any closed surface is equal to \( 4\pi \) times the mass enclosed by that surface.'

For the Solar System \( \psi_m \approx 8\pi * 10^{30} \) and for the Milky Way \( \psi_{mw} \approx 4\pi * 10^{44} \) kg.

1.10 Gravitational Potential

The gravitational potential at a point in a gravitational field is defined as being numerically equal to the work done in bringing a unit mass from infinity where the potential is zero, to that point.

In order to move a mass from one point in a g-field to another, work needs to be done on the mass. The two points must therefore, have some property associated with them which is different at the two points. This property is gravitational potential, symbol \( U \) measured in units of J/kg. If two points have different gravitational potential, then the g-potential energy \( PE \) equal to \( mU \) Joules of a mass \( m \) changes as a result of moving from one point to another. The potential is a property of the field, and the potential energy depends on both the field and the size of the mass.

In Figure 6 a unit mass \( m \) of 1kg is free to move in a gravitational field under the influence of the g-vector. The force it experiences is \( f = g \times 1kg \). Work must be done to move the mass against the gravitational force from A to B, and is called the change in gravitational potential between A and B, units W J/kg. Points A and B are situated in the g-field produced by mass \( M \) at O. As gravitational forces are only attractive the g-vector acts towards the centre of \( M \). Path \( L \) is arbitrary between A and B and P1 and P2 are points on this path which are separated by a small distance \( dl \). The work done by the external force \( F \) in moving the unit mass \( m \) from P1 to P2 is the product of the component of \( F \) in the direction of motion, ie \( f \cos \theta \) in the direction of motion opposite to \( f \). Since \( f = Mg \) and \( M = 1kg \), then the work done (\( W \)) is \( g \cos \theta dl \) and by definition this is the change in potential between P1 and P2. The gravitational potential difference between A and B is found by adding all products of \( g \cos \theta \) with \( dl \) for all the small distances like \( dl \) which go to make up the path \( L \). This is usually written as a line integral as follows:

\[
W = \int_{L} g \cos \theta dl \text{ Joules.} \tag{1.10.1}
\]

The ‘\( L \)’ identifies the path over which the integration is to take place, but the work done can also be expressed as:

\[
W = mv^2 / 2, \tag{1.10.2}
\]

which reduces to:

\[
W = v^2 / 2 \text{ for the unit mass } m, \text{ where } v \text{ is its velocity.} \tag{1.10.3}
\]

The equation for the g-potential at B, \( U_B \) with respect to that of A, \( U_A \) is: \( U_B - U_A = \int_{L} g \cos \theta dl \). \tag{1.10.4}

Since the work done is positive then the change in potential is positive so that the gravitational potential at \( B \) is less negative than that at \( A \). This indicates the potential rises as the unit mass is moved against the direction of the g-field. The unit of \( U \) is J/kg. The U
difference between the two points is 1 J/kg when 1 Joule of work is required to move 1kg of mass from the lower potential at A to the higher at B. From Figure 7, \( g_m = -\frac{GM}{x^2} \).  

We now suggest a suitable Gaussian surface which is spherical of radius x metres. See Figure 7, the centre of this surface is the centre of a spherical mass M. The net flux \( \psi \) across the Gaussian surface at x is equal to the mass enclosed by the surface. 

\[
D_x = \psi / x^2,
\]

But, 

\[
\psi = -M,
\]

so, 

\[
D_x = -M / x^2,
\]

and also, 

\[
D_x = g_x / G.
\]

From 1.10.7 and 1.10.8, 

\[
g_x = -GM / x^2.
\]

The direction of the \( g \)-vector is radially inward from points on the surface of the Gaussian sphere. As Earth is very nearly spherical with radius \( R_e \), its surface can be taken as Gaussian, hence, 

\[
g_e = -GM_e / R_e^2.
\]

Now imagine a spherical surface, centre O and radius x metres, to be drawn through P1. According to Gauss’ theorem the flux \( \psi \), across this surface is equal to the mass enclosed by the surface. 

Hence, 

\[
\int_L G\psi \cos \theta dl / x^2,
\]

\[
= \int_L GM \cos \theta dl / x^2 \text{ Joules.kg}^{-1}.
\]

The limits of integration are at A and B, the end points of the arbitrary path L. At A, \( x = OA \) and at B, \( x = OB \). Therefore, taking all the constants through the integral sign and noting that \( \cos \theta dl = dx \), we have:

\[
U_B - U_A = GM \int_L dx / x^2,
\]

\[
= GM[-1 / x]_AOB^B,
\]

\[
= GM[1 / OA - 1 / OB] \text{ Joules.kg}^{-1}.
\]

Note that the g-potential difference between A and B is independent of the path between the two points and therefore is a scalar quantity. The expression we have obtained for this g-potential difference is for a gravitational field produced by a single point mass. If there are \( n \) point masses we use the Principle of Superposition and this means calculating the g-potential difference between any two points due to each mass considered to be acting alone and then adding the results algebraically. If the point masses are \( M_1, M_2, \ldots M_n \) and the distances from each one of the points A and B are \( d_{A1}, d_{A2}, \ldots d_{An} \) and \( d_{B1}, d_{B2}, \ldots d_{Bn} \) respectively, then the expression is:

\[
U_B - U_A = \sum_{m=1}^{n} GM_m / d_{Bm} - 1 / d_{Am}. \]

1.11 Gravitational Potential Gradient

The two points A and B in Figure 8 are separated by a short distance dx. We have 

\[
U_B - U_A = -\int_L g \cos \theta dx
\]

is the component of \( g \) in the x-direction and is written 

\[g_x\], so that, 

\[
U_B - U_A = -\int_L g_x dx.
\]

If the gravitational potential difference is written as \( \partial U \) then 

\[
g_x = -\partial U / \partial x,
\]

where \( \partial U / \partial x \) is called the g-potential gradient in the x-direction. The differentiation is used as gravitational fields are essentially three dimensional and the g-potential may well vary in the y- and z-directions as well and there will be \( \partial U / \partial y \) and \( \partial U / \partial z \). So in addition we have 

\[
\partial y = \partial U / \partial y \quad \text{and} \quad \partial z = \partial U / \partial z.
\]

1.12 The Gravitational Potential at a Point P Due to n Masses

So far we have discussed g-potential difference, but if we need to state the g-potential ‘at a point’ then we must define a datum, so we take the zero g-potential at infinity. Let us then determine g-potential difference between B and A where A is at infinity, so that it has zero g-potential and assuming no interference from other gravitational bodies which would tend to curve d_infinity.

\[
U_B - U_A = U_B - 0 = \sum_{m=1}^{n} GM_m / d_{Bm} \text{ Joules.kg}^{-1}.
\]

1.13 The g-Potential at a Point

There is no absolute value for the gravitational potential at a point, as it will depend on where the datum for zero is taken. However, taking zero g-potential to be at infinity, then:
**Gravitational potential at a point** is defined as the work required to bring a unit mass from infinity to the point.

We shall now obtain an expression for the g-potential at a point d from the centre of a spherical mass of radius \( r \). If we let the mass of the sphere be \( M \) kg, then, by Gauss’ theorem the g-flux across the gaussian surface, a spherical surface of radius \( x \), will be \( M \).

In Figure 9 the g-flux density at x is,

\[
D_x = \psi / x^2, 
\]

hence,

\[
D_x = M / x^2 \text{ kg.m}^{-2}. 
\]

The g-field strength at x is given by:

\[
g_x = -GM / x^2 
\]

and when \( x=R \), then:

\[
g_e = -GM_e / R_e^2, \text{ the Newton equation.} 
\]

The work required to move a unit of mass (1kg) along an element dx of the path L is:

\[
g = -g \cos \theta \, dx = -g_x \, dx 
\]

The work required to bring a unit of mass from infinity to the point P, therefore, which by definition is the g-potential at P, is given by:

\[
U_p = \int_{x}^{d} g_x \, dx = \int_{x}^{d} GM_e / x = -GM_e / x \text{ J.kg}^{-1}. 
\]

1.14 The g-Potential at a Point Due to Two Massive Bodies

OAA = OB = 1, OP = d, BP = d₁ and AP = d₂.

Figure 10 shows two massive bodies separated by a distance which is small compared with the distance at which the effects are measured. The point P is specified in polar form by \( P = \theta \) from the origin O. Using the expression 1.13.5 together with the Principle of Superposition, we have for the g-potential at point P:

\[
U_p = -GM / d - M_1 / d_1 \text{ J.kg}^{-1}. 
\]

We can simplify this by making approximations, by constructing the perpendiculars from A and B onto DP. Then: \( OC = OD = l \cos \theta \). Since \( d \pi dl \) then \( d_1 \approx d - l \cos \theta \) and \( d_2 \approx d + l \cos \theta \) and substituting these into 1.14.1 we obtain:

\[
U_p = -GM [2l \cos \theta / (d^2 - l^2 \cos^2 \theta)], 
\]

but since \( d \cos \phi \) \( 2l \) then \( d^2 \cos \phi \) \( l^2 \cos^2 \theta \) and \( l^2 \cos^2 \theta \) may be neglected.

\[
U_p = 2ML^2 \cos \theta / d^2, 
\]

where 2ML is called the moment of the two gravitational bodies measured in kg.m.

1.15 The Gravitational Potential at a Point Q Perpendicular Distant from the Centre Line AB

As \( \theta = \pi / 2 \), \( \cos \theta = 0 \) and therefore \( U_p = 0 \) everywhere along the perpendicular QQ’. The line is called the line of gravitation equipotential. If \( M_1 \neq M_2 \) then the perpendicular line of g-potential moves along the line AB towards the lesser mass.

1.16 Surface Gravitational Density

The direction of the \( g \) and \( D \) vectors at all points on an equipotential surface is perpendicular to the surface and towards the centre of mass. The gravitation per unit area (1m²) of the surface is called the **surface gravitational density**, symbol \( \sigma \), for which the unit is \( \text{m}^2 \cdot \text{s}^{-2} \) or \( \text{N.m}^{-2} \). The equipotential surface shown in Figure 11 has a surface gravitational density of \( \sigma \), and the Gaussian surface of area \( dS \) encloses a small part of the equipotential surface. The sides, ad and bc are perpendicular to the surface, so the flux crossing the Gaussian surface is \( \sigma dS \) and all of it crosses the outer surface cd. Now the gravitation enclosed by the Gaussian surface is \( \sigma dS \) and so by Gauss’ theorem the g-flux out of the surface is equal to \( \sigma dS \). The flux crossing the inner surface ab is also \( \sigma dS \) because the g-flux density D inside the mass decreases linearly from \( \sigma dS \) at the surface to Zero at the centre of mass. The g-flux crossing sides ad and bc is zero because D is parallel to them. It follows that all the g-flux must cross cd and that the g-flux density D immediately outside the mass is equal to the surface gravitational density ie D. So \( D = \sigma \), \( g = DG \) and therefore:

\[
g = G \sigma 
\]

**Conclusion**

In Section 1 we introduced the concept of curved space caused by the distortion of the gravitational lines of force that exist between any two masses, and went on to develop basic gravitational theory from first principles. This development shows the importance of Gauss’ theorem for gravitation.
ANOTHER VIEW OF NEWTONIAN GRAVITY by William F Tottle (Part 2)

Introduction
In this Part we continue on from Part 1 and look at various aspects of cosmology with respect to the solar system limiting the number of astral bodies involved. Then go on to develop further the concept of curved space in particular for the Sun, Earth, Moon system, finishing with a graphical representation of the four field gravitational system incorporating the galactic field.

2 Calculation of the Gravitational Quantity \( g \) and Gravitational Force \( f \) for the Three Bodies Under Consideration and of the Milky Way

Sign convention: In Sections 2 and 3, only absolute values are calculated. The direction of these depend on which solar body is the subject and this will be considered in Section 5.

2.1 Earth

\[
\begin{align*}
\text{Mean } \quad g_e &= GM_e / R_e^2 = 9.8 \, \text{m.s}^{-2}. \\
f_e &= g_e M_e = 5.854 \times 10^{25} \, \text{N}.
\end{align*}
\]

2.2 Moon

\[
\begin{align*}
g_m &= GM_m / R_m^2 = 1.623 \, \text{m.s}^{-2}. \\
f_m &= g_m M_m = 1.193 \times 10^{23} \, \text{N}.
\end{align*}
\]

2.3 Sun

\[
\begin{align*}
g_s &= GM_s / R_s^2 = 2.741 \times 10^3 \, \text{m.s}^{-2}. \\
f_s &= g_s M_s = 5.455 \times 10^{32} \, \text{N}.
\end{align*}
\]

2.4 Forces Exerted by one Body on Another in its Orbit

The Law of Universal Gravitation gives this force. We need an estimate for the total equivalent mass of the Milky Way. Text books give various figures for the number of stars in the Milky Way, ranging from \( 1.7 \times 10^{12} \) to over \( 10^{14} \).

The estimated mass of the Milky Way \( M_{mw} \), where \( \omega_{mw} = 2\pi / T_{mw} = 2.008 \times 10^{14} \) secs is the orbital period of the sun, is given by:

\[
M_{mw} = \left( \frac{\omega_{mw}^2 R_{(mw-s)}^3}{G} \right) = \left\{ \frac{4\pi^2 \times (2.365 \times 10^{20})^3}{(2 \times 10^{14})^2 \times 6.7259 \times 10^{-11}} \right\} = 1.941 \times 10^{44} \, \text{kg}
\]

If \( n_{mw} \) is the number of stars estimated to be in the Milky Way, each of Sun mass, then:

\[
n_{mw} = M_{mw} / M_s = 9.754 \times 10^{13} \, \text{stars}.
\]

2.5 The force exerted by Earth on the Moon in its orbit is:

\[
f_{(m-e)} = GM_e M_m / R_{(m-e)}^2 = 1.982 \times 10^{20} \, \text{N}.
\]

2.6 The force exerted by the Sun on the Earth:

\[
f_{(e-s)} = GM_s M_e / R_{(e-s)}^2 = 3.579 \times 10^{22} \, \text{N}.
\]

2.7 The average force exerted by the Sun on the Moon:

\[
f_{(m-s)} = GM_m M_s / R_{(m-s)}^2 = 4.358 \times 10^{20} \, \text{N}.
\]

2.8 The force exerted by the MW on the Earth:

\[
f_{(e-mw)} = GM_e M_{mw} / R_{(e-mw)}^2 = 1.397 \times 10^{18} \, \text{N},
\]

2.9 The force exerted by the MW on the Moon:

\[
f_{(m-mw)} = GM_m M_{mw} / R_{(m-mw)}^2 = 1.702 \times 10^{16} \, \text{N},
\]

2.10 The force exerted by the Mw on the Sun:

\[
f_{(s-mw)} = GM_s M_{mw} / R_{(s-mw)}^2 = 4.608 \times 10^{23} \, \text{N}.
\]

3 The Centripetal Forces Associated with Orbiting Bodies

3.1 For the Moon the centripetal force \( f_{mc} \) is given by:

\[
f_{mc} = M_m \omega_m^2 R_{(e-m)}, \text{ where } \omega_m = 2\pi / T_m = 1.995 \times 10^{20} \, \text{N}.
\]

3.2 The Earth’s centripetal force: \( f_{ec} = M_e \omega_e^2 R_{(e-e)} = 3.542 \times 10^{22} \, \text{N} \).

3.3 The Sun’s centripetal force: \( f_{sc} = M_s \omega_s^2 R_{(s-mw)} = 4.605 \times 10^{23} \, \text{N} \).
4 The Solar System
4.1 The Elliptical Orbit of Earth
Referring to Figure 12, the parametric equation for the ellipse is given by:
\[ x = a \cos \theta, \quad y = b \sin \theta. \]
The radius vector \( \mathbf{R}_{(e)} \) which joins the Earth to the Sun, sweeps out equal areas in equal times. The radius of the ellipse of the ellipse is \( R_{(e)} \), \( E_e \) is the eccentric anomaly and the true anomaly \( \theta \) is the angle swept out by the radius vector since perihelion, \( a_e \) is the length of the semi-major axis and \( b_e \) is the length of the semi-minor axis. The position of Earth in elliptical motion about the Sun can be found by means of \( E_e \) and this requires solving Kepler’s equation of elliptical motion:
\[ E_e = e \sin E_e, \]
where \( e \) is the eccentricity of motion calculated by:
\[ e = \sqrt{1 - \left( \frac{b_e}{a_e} \right)^2}. \]
If \( a_e \) is normalised ie \( a_e = 1 \), then \( b_e = 0.9998603 \), which is useful in solving the equations containing these quantities. Note that non normalised the astronomical unit, equals \( 1.4959787 \times 10^{11} \) metres.
Now M the mean anomaly is found from: \( M = 2\pi / T \), where \( t \) days is the time taken for Earth to rotate through \( \theta \) radians, \( T \) the period of motion = 365.24 days.

Now,
\[ E_e = \tan^{-1} \left[ \frac{b_e \sin \theta}{a_e (\cos \theta - e_e)} \right], \]
and the magnitude of the radius vector is given by:
\[ R_{(e)} = \left( b_e \sin \theta \right)^2 + a_e^2 \left( \cos \theta - e_e \right)^2 \]
or,
\[ R_{(e)} = a_e \left( 1 - e_e^2 \right) / \left( 1 + e_e \cos \theta \right). \]

4.2 The Lengths of Earth’s semi-major axis \( a_e \) and semi-minor axis \( b_e \)
The area of an ellipse formed by the Earth’s rotation about the Sun is given by:
\[ A_{(e)} = \frac{\pi}{4} \left( a_e b_e \right), \]
and as for the Earth we have \( b_e = 0.9998603 a_e \), then:
\[ A_{(e)} = 0.9998603 \pi a_e^2. \]

According to R8, the mean distance of Earth from the Sun is \( R_{(e)} = 1.4959787 \times 10^{11} \) metres so \( A_{(e)} \) must equal the area of a circle of this radius. Hence:
\[ 0.9998603 \pi a_e^2 = \left( 1.4959787 \times 10^{11} \right)^2 \pi, \]
from which,
\[ a_e = 1.4960832 \times 10^{11} \text{ m}, \]
and,
\[ b_e = 1.4968955 \times 10^{11} \text{ m}. \]

4.3 Table of Results for the Elliptical Motion of Earth (angles in radians)
Where \( \theta = \rho - \delta \rho \), where \( \rho \) is the true anomaly, and \( \delta \rho = 6\pi GM_e / \{ c^2 a_e (1 - e_e^2) \} \) is the perihelion precession angle.
\[ E = \tan^{-1} \left[ b_e \sin \theta / \{ a_e (\cos \theta - e_e) \} \right] \quad M = E - e \sin E \quad t = MT / (2\pi) \]
\[ R_{(e)} = \left[ (b_e \sin \theta)^2 + a_e^2 (\cos \theta - e_e)^2 \right]^{1/2}, \]

<table>
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<tr>
<th>( \theta )</th>
<th>( E )</th>
<th>( M )</th>
<th>( t )</th>
<th>( R_{(e)} )</th>
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<td>0,2\pi</td>
<td>0,2\pi</td>
<td>0,365,24days</td>
<td>1.4711010*10^{11} metres</td>
</tr>
<tr>
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<td>0.7986352</td>
<td>0.7984022</td>
<td>46.4109</td>
<td>1.4711032*10^{11}</td>
</tr>
<tr>
<td>( \pi/2 )</td>
<td>1.5972679</td>
<td>1.596802</td>
<td>92.8217</td>
<td>1.4711102*10^{11}</td>
</tr>
<tr>
<td>3( \pi/4 )</td>
<td>2.3958954</td>
<td>2.3951968</td>
<td>139.2319</td>
<td>1.4711220*10^{11}</td>
</tr>
<tr>
<td>( \pi )</td>
<td>( \pi )</td>
<td>( \pi )</td>
<td>182.62</td>
<td>1.5211080*10^{11}</td>
</tr>
<tr>
<td>3( \pi/2 )</td>
<td>4.7991722</td>
<td>4.792157</td>
<td>278.5669</td>
<td>1.4711854*10^{11}</td>
</tr>
</tbody>
</table>

4.4 Calculation of the Precession of the Perihelion of Mercury
R5, p198
The magnitude of Mercury’s radius vector where \( \theta = \phi + \delta \phi \) is given by:
\[ R_{(me-s)} = \left[ (b_{me} \sin \theta)^2 + a_{me}^2 (\cos \theta - e_{me})^2 \right]^{1/2}, \]
The values of the constants are: \( a_{me} = 5.848104 \times 10^{10} \text{m}, \ b_{me} = 5.729358 \times 10^{10} \text{m} \) and \( e_{me} = 0.206004 \) and the angle of precession can be calculated from:

\[
\delta \theta = 6\pi GM_e / \{ a_{me} c^2 (1 - e_{me}^2) \} = 4.9730684 \times 10^{-7} \text{ rads per orbit},
\]

\[
= 42.570579 \text{ arc secs per century}.
\]

4.5 The precession of the perihelion of Earth

\[
\delta \phi_e = 6\pi GM_e / \{ a_e c^2 (1 - e_e^2) \} = 1.861944 \times 10^{-7} \text{ rads per orbit},
\]

\[
= 3.838 \text{ arc secs per century}.
\]

At this rate of precession it would take Earth 337,675 centuries to complete just on anomalistic year. It is not surprising then, this has been ignored and can continue to be so.

4.6 The Barycentre of the Earth-Moon System R6

In Figure 13 the Earth-Moon system at perigee shows the barycentre - its centre of gravity, marked B on the line joining the centres and within the Earth. Let its distance from Earth’s centre be \( d_1 \) the \( d_2 \) is its distance from the Moon’s centre, where \( d_2 = R_{(e-m)} - d_1 \). Taking moments about B we have:

\[
M_e d_1 = M_m d_2,
\]

from which we derive that:

\[
d_1 = M_m R_{(e-m)} / (M_e + M_m),
\]

\[
= 4.6243928 \times 10^6 \text{ metres}.
\]

Relative to both Earth and the Moon, B’s position varies proportionally to the distance between them and at perigee \( d_1 \) is 4.305137 \times 10^6 metres and at apogee it is 4.8782719 \times 10^6 metres. As B must always lie on the line joining the centres of the two bodies, then Earth must rotate elliptically about B. The locus of the Earth’s centre is shown as a dotted ellipse with the cardinal angles of \( \phi \) ie 0, \( \pi / 2 \), \( \pi \), and \( 3\pi / 2 \), the Moon’s eccentric angle shown.

4.7 The Inclination of the Moon’s Orbit R6

Another phenomenon is that the Moon’s orbit inclines plus or minus 5.145396º to the plane in which the Earth’s orbit lies. This is called the elliptic, the plane in which the barycentre of the Earth-Moon system, the BEM, orbits the Sun. The inclination of the Moon’s orbit adds to or subtracts from the inclination of the Earth’s rotational axis causing declination of the Moon. The declination when observed from Earth varies between plus or minus 28.5º when the two inclinations add, and between plus and minus 18º during the period of opposition, with the maxima and minima of declination repeating every 18.6 years, the period in which the ascending node of the Moon’s orbit precesses through a complete circle. R6

4.8 Forces Acting at a Point in the Sun–Earth-Moon System

Any calculation of these forces will only be approximate as they will not include the influence of the other planets in the solar system, but should be a good approximation. If we stop the Sun the results will depend on the relative positions of the Earth-Moon system as it rotates around the Sun.
Figure 14 shows the Sun at the origin of a cartesian co-ordinate system in two dimensions. The Moon is omitted due to the scale. The elliptical orbit of Earth is shown almost circular with Earth at point $(O,b)$. Now visualise the position of the Moon in the half-moon position a little to the right of Earth. The line of equi-force for the Earth-Sun is shown parallel to the x-axis and distant $d_{O,P2}$ from it. Because of the Moon’s Position the major line of gravitational force from Earth to the Sun will not be linear, but distorted slightly in the form of two arcs of circles in tandem. The larger arc from the Sun to point $P$ on the line of equi-force has its centre at $C_1$ and the smaller arc from $P$ to $b$ has its centre at $C_2$. Both $C_1$ and $C_2$ are on the line of equi-force which is the line of all such centres. These arcs are shown grossly distorted in magnitude in order to demonstrate clearly the distortion of the line of force. Only the major (shortest) line has been shown for clarity. Imagine now the Moon rotating to a position of lunar eclipse ie on the y-axis immediately above Earth in the Figure. The major line of force will now be linear as the forces involved all act in the same straight line. As the Moon rotates to this new position the lengths of the arcs reduce, becoming the least curves and therefore with minimum distance at lunar eclipse. Also $C_1$ moves further out along the line of equi-force until at lunar eclipse $C_1$ is at its maximum distance. $C_2$ changes its position similarly. This shows that gravitational lines of force are for the most part (perhaps always) curves, constantly changing in length to some degree depending on the forces influencing the situation significantly. This phenomenon is paramount throughout the universe, which makes it extremely difficult for experimenters to ascertain exact values for $G, g,$ and other constants because they are close to being ‘constant’ when measured but not quite.

Now $M_e$ is a constant, but any measurements taken to determine the Earth’s mass should always be by an observer in the direct line of a solar eclipse say. Only in this way will the data be consistent enough to produce accurate results. In Figure 14 the forces acting on the Sun can be resolved into two components acting along the axes. The first along the y-axis is given by:

$$f_y = f_{(m-o)} + f_{(e-o)} \sin \theta + f_{(m-e)} \cos \phi.$$ 

where $\theta$ is the Earth’s eccentric angle and $\phi$ the Moon’s eccentric angle.

$$f_x = f_{(m-o)} + f_{(e-o)} \cos \theta - f_{(m-e)} \sin \phi.$$ 

So the instantaneous magnitude of the resultant force is:

$$f_r = \sqrt{(f_{(m-o)} + f_{(e-o)} \sin \theta + f_{(m-e)} \cos \phi)^2 + (f_{(m-o)} + f_{(e-o)} \cos \theta - f_{(m-e)} \sin \phi)^2},$$

where the component forces are those computed from the Newtonian equations.

In polar form the argument of the resultant force is:

$$\theta_r = \tan^{-1}\left(\{f_{(m-o)} + f_{(e-o)} \sin \theta + f_{(m-e)} \cos \phi\} / \{f_{(m-o)} + f_{(e-o)} \cos \theta - f_{(m-e)} \sin \phi\}\right),$$

For the position of Earth in Figure 14 with a half-moon in the first quadrant $\cos \theta = 0, \sin \theta = 1, \cos \phi = 0, \sin \phi = 1$, so:

$$f_r = 3.5896547 \times 10^{22} \text{ N}$$

and the argument is:

$$\theta_r = -1.5652749 \text{ rads (90.316356°)}.$$ 

Note: the resultant force acts toward the Sun, in keeping with the usual formulae for Earth’s gravitation which carry the negative sign, we show the force acting towards the Sun as positive. We can now represent this force on the diagram and the line OQ is the tangent at O to the arc OP of the circle. The tangent OQ is perpendicular to the radius r of this circle and the angle subtended at $C_1$ onto the arc OQ is $\alpha = 5.5214538 \times 10^{-3}$ rads (0.316356°). OQ approximately equals the length of the semi-minor axis $b$, so,

$$r = b / \tan \alpha = 2.7092145 \times 10^{15} \text{ metres.}$$
Referring to Figure 15, the length of the chord OP is:

\[ OP = 2r \sin(\alpha/2) = 1.4958784 \times 10^{11} \text{ metres.} \]  

In Figure 16, the distance \( d_{(e-P_2)} = b - d_{(e-P_1)} = 1.495891 \times 10^{11} \) m, so,

\[ PP_2 = OP \sin(\alpha/2) = 4.1297065 \times 10^8 \text{ m}, \]  
and,

\[ \beta = \tan^{-1}\left(d_{(e-P_2)}/(PP_2)\right) = 1.0985738 \times 10^{-3} \text{ rads (0.0629436°)}. \]  

Now,

\[ QP = \left[d_{(e-P_2)^2} + (PP_2)^2\right]/2 = 2.0648545 \times 10^8 \text{ m.} \]  

Referring to Figure 17 the radius \( r_P \) of the smaller arc is equal to the length RP:

\[ R_2 = QP/\cos\beta = 2.0648557 \times 10^8 \text{ m}, \]  
and

\[ \gamma = \pi/2 - \beta = 1.5696978 \text{ rads (89.937056°)}. \]  

Angle of arc \( bP \), \( 2\gamma = 3.1393955 \text{ rads (179.87411°)}. \)

Length of arc \( bP = 2r_2\gamma = 6.4823988 \times 10^8 \text{ m} \)

Total curved distance \( Ob = bP + r\alpha = 1.5023627 \times 10^{11} \text{ m}, \) so the extra distance equals: \( \text{curved} \ Ob - b = 6.4671988 \times 10^8 \text{ m}, \) which is the maximum extra distance of the major line of gravitational force. As force is directly proportional to gravitational acceleration then \( g \) must also vary, and any measurement of the Universal Constant \( G \) will also vary, which is shown in the different results obtained in Russia, Germany and New Zealand in recent measurements of \( G. \)

5 The Four-Field System of Gravitational Fields

Figure 18 shows a representation of the 4-field system of solar gravitation. The forces of the Milky Way acting on the Sun, Earth and the Moon are such that each of the three bodies is contained and in equilibrium. As the forces are based on attraction the forces between each pair are shown acting from the smaller to the larger body. Several observations can be made from this simple diagram.

(a) All the lines representing the gravitational fields and the lines joining the points of equal force, are curves. Hence, Newton’s Laws of Gravitation can only be approximate as these are based on forces acting in straight lines, although close enough to yield results within an accepted tolerance and therefore suitable for most practical purposes. This new theory predicts the Newtonian forces will be slightly greater than in reality due to it being based on linear distances and scientific measurement of the difference is one way to help verify this theory. The longer the distances between the two bodies the closer the two theories become.

(b) The galactic field compresses to some degree the fields of the solar bodies thus increasing their flux density near the bodies.

(c) Redrawing the galactic field from another direction does not change the situation. The disposition of the galactic field is difficult to predict as the solar system is surrounded by galactic stars.
The gravitational system is in fine balance. Any tendency for the Moon to move closer to Earth due to an increase in $f_{mc}$ would immediately be cancelled by an increase in $f_{m-e}$, which restores the Moon to its natural orbital radius which depends on its angular velocity.

The curvature of ‘space-time’, which in fact is proportional to the resultant gravitational forces acting in that locality, can be clearly seen.

6 Conclusion

Certainly, Newton’s theory is extremely close to reality as to render this theory redundant for most practical purposes. Never the less, readers with inquisitive minds may wish to pursue the exploration with the author. Einstein’s theory uses Reimann geometry which is difficult to understand and not for the average person, so it is hoped that this theory with all its faults, and there are probably many, can be developed to produce a natural and simple theory which is easier to understand. Constructive criticism is welcome the other kind is not, so if a reader has something to contribute such as corrections to calculations and/or theory, additions where the content is weak, misspellings and typing errors or advice on where to go next the author will welcome it. Please send contributions to:

bill@tottle250.freeserve.co.uk, acknowledgements will be given to all e-mail received where help is given.

References

R1 Adapted from ‘Emission-Absorption-Scattering (EAS) Sub-Quantum Physics’ by R S Fritzius, Intro.
R3 Hutchinson Encyclopedia, Helicon, ISBN 1-68986-333-7
R6 Inconstant Moon, www.fourmilab.ch/earthview/moon_ap-per.html
R7 Oxford Interactive Encyclopedia, www.learning.co.com

Table of Constants and Variables Used & Calculated

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<thead>
<tr>
<th>G = 6.67259*10^{-11} Nm^2kg^{-2} or m^3kg^{-1}s^{-2}</th>
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