SOURCE-FREE MAGNETIC STRUCTURES

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Abstract

A seminal discovery shaping science and technology thus far is that of electromagnetic wave by James Clerk Maxwell in the 1860s, based on what is today known as Maxwell's equations. It is possible that misinterpretations of these equations by his immediate successors have been promulgated ever since, preempting a concomitant development: The source-free static magnetic field structure as a co-equal consequence of the same equations. The entire concept of magnets and magnetic field – one of the oldest chapters of physics - may now need reconsideration, with broad and deep implications for both science and technology. The new result may bring to confluence a number of disparate concepts of physics.

KEY WORDS: magnetic field; string theory; fusion energy; dark matter; dark energy; gravitation; empty space; cosmology; future technology.

I. THE HYPOTHESIS

The recent paper, *The missed physics of source-free magnetic field* [1], attempts to elicit a deeper and wider foundation of physics than is known today: a greater meaning of magnetic field beyond the conceptions of James Clerk Maxwell and Albert Einstein. In simple terms, the paper proves that the present view of static magnetic field as linked to a source (lodestone, permanent magnet, steady current) is only a special case; that magnetic field at first is not a source-dependent concept. However, Maxwell's equations are fully consistent with this expanded meaning. Furthermore, a 1996 paper [2], *Gravitational mass of magnetostatic field*, proposed another fundamental result: that magnetic field in empty space is a matterless mass that is not encompassed in Einstein's mass-energy relation, and that blurs the distinction between mass and energy. By combining these two new results, Ref. [1] describes a particle made entirely of source-free static magnetic field, having a finite mass. It may be a particle that already exists in nature, being for example the ultimate (smallest) constituent of matter. Additionally, or alternatively, it may be one that can be created in the laboratory for technology applications.

To gain an initial sense of what is being discussed, visualize the familiar bar magnet with its invisible magnetic field structure around it. Next imagine that the bar itself is gone, but the invisible structure remains in place. Then imagine that this invisible structure has a mass, and does everything that a mass does (moves, accelerates, falls under gravity, etc). This is what one has here except that the geometry of the structure is now more ramified. In shorthand, call such a structure SFMS (Source-free magnetic structure).

II. A CONTEXT IN ELECTROMAGNETIC THEORY

Electromagnetic (EM) Theory is one area of physics where certain fundamental issues remain unresolved to this day [3]. Of interest here are issues connected to the discoveries of the EM Theorist Hannes Alfvén [4]. Alfvén's ideas were so revisionist at the time he proposed them that contemporary experts rejected them outright. They criticized him for violating the textbook theorem that there can be no electric field inside an infinitely conducting medium.

As is well known today, Alfvén showed that EM waves traveling in conducting fluids become entangled with the fluid and thus slowed down – with the fluid itself co-moving in a wavy motion. His result – the Alfvén wave - owed itself to the fluid being conducting. It was shown subsequently that this restriction did not apply, and that the effect was universal - present also in the remaining class of fluids, dielectrics. It has been shown that there exist generalized expressions that reduce to conducting medium or dielectric medium result in the appropriate limit (see e.g., [5], and earlier references therein). This generalization may be a needful development, not because it has any practical value, but because it seems to point to an unseen portal behind which wait yet unconsidered ideas.

The above development meant that Alfvén waves pertain to any fluid media. That being so, the question arose: Is there then an underlying general phenomenon applicable also to empty space? Reference [6] showed that there indeed is, and described the related wave phenomenon in empty space as a *companion wave*. This result logically led to the further conclusion that static magnetic field in empty space is a mass [2]. That being the case, the further conceptual questions arose: If this field is a mass, why should it be dependent on a source for its manifestation? Why should a mass be entirely "parasitic"? Furthermore, using the concept of companion waves, it was also shown that the energy in a static magnetic field can be drawn down locally in a region far from the source of the field, without prior communication with the source [7]. This raised more questions about the inevitability of the source. Following up on these questions, one is logically led to the source-free static magnetic field structures [1]. Such structures, however, are today considered forbidden by a textbook theorem.

III. ON CONTRAVENING A TEXTBOOK THEOREM

It has been said somewhere that those who see the invisible can do the impossible. Maxwell's revision of a textbook theorem (the Ampère's theorem), and the resultant conception of EM wave beautifully illustrate this point. His simple way of thinking about invisible happenings in empty space by conjuring up in his mind elaborate moving mechanical structures with rack-and-pinions, cogs, wheels etc [8] is a case in point. Had he the time, he would likely have arrived at the idea of source-free magnetic structures in his own way. But Maxwell died at a relatively early age. Following him, there appears to have prevailed a sense that he had left some fundamental work undone. This sense is conveyed by the fervor with which many leading EM theorists flocked to this area in the decades following the experimental verification of EM (radio) waves in 1890 – among them, Einstein, who started delving into the issue (see, e. g., [9]). But the proceedings there seem to have become deeply mired in acrimonious debate [3], and he seems to have wandered away from the subject. The unfinished fundamental work – if there was one - was never successfully addressed.

Was the source-free structure this fundamental work? If it was, the idea was not to see the light of day. Somewhere along the line experts created a textbook theorem – essentially a roadblock in front of the said portal - declaring this structure forbidden [1]. After Einstein, Alfvén may have been the person most likely to have addressed Maxwell's unfinished work. Like Maxwell, Alfvén had revised the existing EM Theory. The latter's personal style of thinking by mental visualization paralleled the former's (Compare Alfvén's frozen flow, magnetic flux ropes etc with Maxwell's cogs, wheels and so on). One can make a case that Alfvén had even advanced in the right direction – with his ideas of free-floating magnetized plasma ring and the force-free magnetic field structure [10]. These ideas may be only one ideational step away from the source-free structure.

But Alfvén, his contribution rejected by experts, wandered away from the basic questions of EM Theory to astrophysical/cosmological questions. Here again, his conception of an interaction between a static magnetic field and a neutral gas moving across it (see e. g., [11]) – contrary to the textbook - may have contained some hint that the nature of magnetic field had not been fully understood. The above thread of history can explain how a foundation of physics may have remained unnoticed and unmissed to date: Time and again, the very people who had the intellectual wherewithal to advance knowledge beyond the perimeter of the current textbook departed the scene. Once having appreciated this, someone curious might circumvent the roadblock of the present textbook theorem, and look to see what lies beyond.

One would be remiss here not to note that an attempt to do perhaps precisely this has been made by Alfvén's protégé, Bo Lehnert [12, 13]. He seems to be guided by a sense of a basic deficiency in EM Theory, and seeks to redress it through an extended formulation of Maxwell's equations. The present study on the other hand seeks to extend the meaning of magnetic field, leaving Maxwell's equations intact.

IV. A HIDDEN PATTERN

IV.1 The useful content in a discarded solution

In brief, this is how the SFMS is derived [1]. First, one solves Maxwell's equations in empty space to derive unrealistic (unphysical) magnetic field structures where the magnetic field becomes infinitely strong towards infinite distance in the –z direction, and becomes infinitely weak in the +z direction (of a cylindrical coordinate system [r, z, φ]). In the conventional EM Theory, this solution is discarded. Instead, one cuts off the structure at the z = 0 plane, and discards the bottom part. The top half remains. One then takes another such half, inverts it, and attaches it to the bottom of the first half. Now there is a complete structure (A1) where the field goes to infinitely weak values both in the +z and –z directions. However, the result of this "surgical" operation is to create a circular sheet current system I_ϕ in the z = 0 plane. Furthermore, even though the field declines towards infinite distances in the radial direction, it does not decline rapidly enough, causing the structure to have an infinite energy.

IV.2 Mathematical definition of the SFMS

As a separate problem, one takes this current I_ϕ and derives the conventional magnetic field structure (A2) due to this current. This also is an infinite energy structure. One then subtracts A2 from A1. This causes the current I_ϕ to vanish identically. It is then shown that the structures A1 and A2 are unequal, so that the subtraction leaves a net magnetic field structure with no currents. This is the source-free magnetic structure, described by the vector potential:

$$\mathbf{A}_\phi(r,z) = b_o \int_o^{\infty} [F_1(R) - F_2(R)]\, J_1(\alpha R)\, dR\, \mathbf{a}_\phi \qquad (1)$$

with

$$F_1(R) = [J_1(\alpha r)/\alpha][R^2 / (R^2+z^2)^{3/2}], \qquad (2)$$

$$F_2(R) = [(2-k^2)K(k) - 2E(k)]R/[\pi k(rR)^{1/2}]. \qquad (3)$$

Here, $k^2 = 4rR/(R^2 + r^2 + z^2 + 2rR)$, and K(k) and E(k) are the complete elliptical integrals of the First and the Second kind; J_1 is the Bessel function; and α is an inverse length parameter. From this, the SFMS magnetic field **b**(r,z) can be calculated.

IV.3 Hidden in the folds of infinity

This field **b** declines in the radial direction fast enough so that this is a finite energy structure. In other words, an undulating (spatially oscillating), finite-energy structure has materialized from superimposing two undulating, infinite-energy structures – the way one might reveal a finite, hidden pattern by superimposing two viewgraphs of seemingly identical patterns that extend to infinity. It is as though the SFMS has so far been hidden, or camouflaged, in the folds of infinity.

V. THE SFMS AS A WAVE

There are two distinct ways to link the SFMS and EM waves: The SFMS itself seen as EM waves; and the SFMS transformable to EM waves (and *vice versa*).

V.1 The wavelike character of the SFMS

The first link is best discussed by invoking a stationary electric dipole antenna in a (X, Y, Z) coordinate system. While a fully generalized discussion of this problem is possible, a simplified geometry is assumed for the present conceptual discussion. Suppose that the elements of the antenna are on the Y-axis, and that the SFMS is traveling with a velocity v along the X-axis, with its own axis being parallel to the Z-axis. Suppose further that the length L of the elements is much smaller than $1/\alpha$. Then the antenna sees a time-varying magnetic field:

$$\mathbf{b}_t(t) = \mathbf{b}(v[|t - t_o|], 0) \qquad (4)$$

and a time-varying electric field:

$$\mathbf{E}_t(t) = \mathbf{v} \times \mathbf{b}(v[|t - t_o|], 0), \qquad (5)$$

t_o being the time instant when the center of the SFMS crosses the origin. This will be interpreted as (or mistaken for) an electromagnetic pulse. It is thus interpreted as a superposition of a large number of electromagnetic wave fields $\mathbf{E}_\omega(\omega)$, where ω is the angular frequency of the wave. Thus – through this example - we have a formal connection between the SFMS traveling at a velocity v and EM waves traveling at a velocity c, the velocity of light:

$$\mathbf{E}_\omega(\omega) = (1/2\pi)^{1/2} \int_o^{\propto} \mathbf{v} \times \mathbf{b}(v[|t - t_o|], 0)\, e^{i\omega t}\, dt \qquad (6)$$

V.2 Similarities and differences

A continuous stream of such SFMSs thus has properties similar to continuous EM wave flow. Let us call this stream SFMW (SFMS wave).

The similarities of the SFMW to the EM wave are:

(1) The SFMW can carry information in a variety of modes, analogous to amplitude, frequency, phase and polarization modulation of EM wave;
(2) SFMW can carry energy;

The differences are:

(1) The SFMW travels at selectable speeds less than c, the velocity of light;
(2) The path of the SFMW is influenced by gravity;
(3) The SFMW travels without spreading of the energy;
(4) An SFMW, depending on its speed, can travel clear through a metal sheet or other surfaces that block EM wave.

The questions that need to be studied with regard to the comparison of an SFMW and an EM wave are:

(1) The attenuation of SFMS in a material medium as a function of its velocity;

(2) Reflection/refraction characteristics/conditions of the SFMS stream.
(3) Interaction between two SFMSs in empty space;
(4) Analogous today's extensive antenna technology [14], it may be possible to develop a SFMW antenna technology. This may help alleviate certain problems related to fundamental limitations of conventional antennas [15].

V.3 SFMS-EM Wave transitions

In terms of consequences of Maxwell's equations, and intuitively, the SFMS and EM wave together should be seen as a whole. The true link between the two, i.e., the transitions

$$\text{SFMS} \leftrightarrow \text{EM wave}$$

can be explored in different ways. If, for example, the SFMS can be shown to be the zero velocity limit of EM wave (or the EM wave the high speed limit of the SFMS), does this hint at a link between the SFMS and the photon?

VI. THE SFMS AS A PARTICLE

VI.1 The particle-like character of the SFMS

The matterless SFMS can be compared to material particles such as an electron. The former has a mass [2]:

$$m = 2\pi\varepsilon_o \int_{-\infty}^{\infty}\int_{o}^{\infty} b^2\, r\, dr\, dz\,, \qquad (7)$$

and a length parameter that may be likened to the de Broglie wavelength:

$$L \sim 1/\alpha\,, \qquad (8)$$

where ε_o is the dielectric permittivity of free space.

The Lorentz transformation of the SFMS is obtained from the following relations for the components of **b** parallel and perpendicular to the velocity v:

$$\mathbf{b}_{\parallel}' = \mathbf{b}_{\parallel} \qquad (9)$$

$$\mathbf{b}_{\perp}' = \mathbf{b}_{\perp}/[\,1 - v^2/c^2\,]^{1/2}. \qquad (10)$$

Thus, except when v ~ c, there is very little distortion of the SFMS in the moving coordinate system. Using the above relation in Eq. (6) one obtains, for v « c,

$$m' = m. \qquad (11)$$

Therefore the SFMS is not in conflict with Lorentz transformation. As discussed in the previous section, what happens when v approaches c is subject of further investigation.

VI.2 Similarities and differences

The SFMS has the following similarities to an electron:

(1) It can flow from one point to another under the influence of EM fields;
(2) It has directionality and sense (Cf. electron spin);
(3) A spinning SFMS would have an observable electric field: a spinning electron a magnetic field;

The differences with an electron are:

(1) SFMS may pass through some barriers that electrons would not;
(2) A SFMS responds primarily to a magnetic field while an electron responds primarily to an electric field.

VII. THE SFMS AS A THIRD ENTITY

The preceding discussion of SFMS observable as EM waves needs to be qualified. From Eq. (6), one would conclude that the corresponding magnetic field of the EM wave is:

$$\mathbf{B}_\omega (\omega) = (1/2\pi)^{1/2} (1/c) \int_o^\infty \mathbf{v} \times \mathbf{b}(v[|t - t_o|], 0)\, e^{i\omega t}\, dt \qquad (12)$$

However, from Eq. (5), one would obtain the Fourier components:

$$\mathbf{b}_\omega (\omega) = (1/2\pi)^{1/2} \int_o^\infty \mathbf{b}(v[|t - t_o|], 0)\, e^{i\omega t}\, dt \qquad (13)$$

Clearly, these differ by a factor v/c. Therefore, while a traveling SFMS can serve many practical functions of EM waves, it is not EM wave.

By conventional definition of magnetostatic energy, the SFMS represents a packet of energy:

$$\epsilon = \pi/\mu_o \int_{-\infty}^{\infty}\int_o^{\infty} b^2\, r\, dr\, dz \qquad (14)$$

where μ_o is the magnetic permeability of free space. Comparing this with Eq. (7), one obtains

$$\epsilon = m c^2/2, \quad (15)$$

which is not in accord with the mass-energy relation for a particle. Furthermore, as v approaches c, a particle with a mass m approaches infinite mass. However, the SFMS under this condition is expected to transform to EM waves, and therefore approach zero mass. Therefore, there are problems with defining the SFMS as a particle.

This gives rise to the possibility that the SFMS is neither a particle nor a wave, but a third type of entity which has not yet a name. Such an entity may have the properties of divisibility, fusibility, and creation *ex nihilo* (see below).

VIII. EXPERIMENTAL VERIFIABILITY

The problem of experimental verification of the SFMS may be considered in three parts:

VIII.1 Verification of a co-located mass

This problem is made difficult by the word "co-located" [2]. When for instance, a bar is magnetized and weighed before and after magnetization, one should detect a change in the weight in accordance with the mass-energy relation. This relation says that the mass *of the bar* increases. Therefore, this experiment is inconclusive as to there being a mass residing in the empty space away from the bar where part of the magnetic field is. So, what type of experiment can make this distinction?

One approach may be based on determining by measurement the moment of inertia I. This quantity for any object is dependent not only on its mass, but also how this mass is distributed. Thus, one can consider a current loop (not necessarily circular), which is spinning about an axis through its center. It is observed with a current flowing in it, and without any current. In the conventional picture, when the loop is energized, its moment of inertia will increase by a calculable amount ΔI due to the increase in the mass *of the loop*. However, in the present picture where the extra mass is not all in the loop, but in the empty space around it, there should be an increase $\Delta I' \neq \Delta I$.

VIII.2 Creation

One approach to creating the SFMS is to begin with a "precursor" structure – e.g. a source-based structure that is closely related to the former. The technique envisioned here may be summarized as follows:

Step 1 – design an electrical circuit for creating a precursor structure.
Step 2 – determine the inductive time constant τ of the circuit.
Step 3 – innovate a technique for interruption of the steady current non-explosively (i.e. without arcing/sparking), and on a time-scale $T \ll \tau$.

The idea here is that in Step 3, the current is shut off, without allowing the inductive energy to either reenter the battery, or to undergo resistive or explosive dissipation. This will want to leave the magnetic energy in place – which is not possible unless this energy could assume the form of SFMS.

As to the precursor structure, the sheet current I_ϕ described in Ref. [1] is the current one might wish to simulate in Step 1. However, this current corresponds to a structure with infinite energy. Therefore, the current has to be modified somewhat (to fall off faster with radial distance) over its theoretical expression.

VIII.3 Detection

Initially, the detection of the SFMS is a problem of detection of magnetic field, requiring an appropriate magnetometer. At the next level, it is the problem of mapping a magnetic field distribution. An additional factor arises from the theoretical result that the SFMS has a mass. This means that upon creation at a high place, the SFMS will fall towards the laboratory floor under gravity. Thus, a series of detectors may be arranged vertically so as to able to register sequentially the passage of a SFMS, with the passage times being in accordance with the acceleration due to gravity. This will also verify that the SFMS has a *gravitational* mass.

IX. POSSIBLE SIGNIFICANCE: THEORY

Some examples of the theoretical issues arising are:

IX.1 A blurring of mass and energy

The SFMS clearly blurs the distinction between mass and energy. The mass-energy relation deals with conversion of mass to energy and vice versa. Symbolically

mass $\leftrightarrow$ energy.

In this definition, a particle and an antiparticle may annihilate to become pure energy, or a particle may be accelerated to increase its mass. However, the SFMS is mass and energy at the same time. Symbolically

mass $\equiv$ energy.

It is a pure mass (like the electron rest mass) *while* it is pure energy (like an EM pulse traveling in vacuum). The SFMS therefore does not a priori come under the purview of the mass-energy relation. Yet if both results are correct, there must be a way to reconcile the two. Exploring this question may lead one to uncharted territories.

IX.2 SFMS and Quantum Theory

The SFMS is simultaneously a particle, a field, and a wave – all in a classical physics sense. One can choose to probe it as a mass, as a field or as a wave. The length $L \sim 1/\alpha$

is similar to the de Broglie wavelength. Thus, the SFMS may have relevance to the area of transition between classical and quantum physics.

IX.3 SFMS and Gravitation

If the hypothesis that the SFMS has a gravitational mass is correct, then this offers a strong constraint on the possible theories of gravitation: A matterless, purely EM entity exerts a gravitational force (or is attracted by that force). This would cast doubt on any theories of gravitation that rely on non-EM phenomena.

IX.4 Possible primacy of magnetic field

The existence of source-free magnetic field might raise the question as to whether or not magnetic field has a primacy over electric field. Yet another area to consider: Does the SFMS shed new light on the old question of existence/nonexistence of the magnetic monopole. It may be that the whole issue of the monopole was but a symptom of the missed physics of source-free magnetic field.

IX.5 String Theory/Particle Theory

The String Theory, created to explore the ultimate makeup of matter, may turn out to be a needless enterprise - resulted from an inability to trace that particular quest to a missing foundation of physics, and so to restore that foundation. A confirmation of the SFMS hypothesis would mean that rudimentary physics has been passed up here to create ersatz physics.

IX.6 Dark Matter and Dark Energy

If SFMSs permeated some parts of the universe, then they would not be “visually” observable, but their mass/energy would be indirectly observable. Even if they were seen in EM wave measuring experiments, they would have been interpreted as EM waves. Thus SFMS might be an area where to look to account for dark matter and dark energy in the universe.

IX.7 Possible creation ex nihilo

Two identical SFMSs, when superimposed with opposing polarity, cancel each other exactly, leaving nothing. It is as though they disappear into the fabric of empty space. One might ask if such particles reappear from out of this space, or *ex nihilo*. Can mass/energy be somehow “suspended” during such transitions? Symbolically, do transitions

SFMS pair ↔ vacuum

connect the physical universe to empty space? If so, how would one reconcile this with the conservation laws?

X. POSSIBLE SIGNIFICANCE: APPLICATIONS

Some examples of the relevant areas of technology are:

X.1 Free space communication

Since SFMWs travel without space loss (geometric spreading) or attenuation, they provide an alternative, energy-saving, means of communication between two link-aligned stations when the travel time is not a practical consideration. It should be borne in mind that the path of SFMSs is influenced by gravity.

X.2 Undersea communication

Long-distance undersea communication today remains a largely unsolved problem. Since the SFMW may propagate with no space loss, and less attenuation than EM waves (depending on the former's speed), they may provide suitable means of undersea communication between submarines; a submarine and a ship; and even a submarine and a shore base.

X.3 Invisible cable

Consider a row of SFMSs, laid out end-to-end in empty space. If one of these is slightly perturbed at a point A, a restoring force will try to bring it back to alignment, and a wave will travel along the row. At some distant point B in the row, this wave can be detected. Hence, communication can occur between A and B through this invisible cable, without any third parties not located on the row being able to receive this communication. Such a structure need not be one-dimensional.

X.4 Directed energy beam

The SFMW may also be launched as a directed, non-spreading beam of energy. A single SFMS may be "fired" (as a bullet, but traveling much faster). One might imagine, for example, a tubular region of graded magnetic field as a SFMS gun.

X.5 Fusion Energy

An example of the possible areas of application in future technology may be that fusion energy research – whose main scientific basis is plasma theory - has lacked the full benefit of essential physics. One of the most important issues in this field is the instability of magnetized or magnetically confined plasmas, causing the magnetic field to assume new (undesirable) configurations. The SFMS is a candidate for such configurations. It is conceivable that a magnetically confined plasma at the threshold of fusion will find a way to disintegrate into SFMSs (directly or via the intermediacy of precursor states such as force-free structures). This might make sustained magnetically confined fusion a theoretical improbability.

XI. REMARKS

The preceding discussion suggests that the SFMS may bring together within it a number of disparate concepts of physics: wave and particle; mass and energy; quantum and classical behavior; electromagnetism and gravitation; physical universe and empty space.

The history of magnets may date back to 2700 BC when Huang Di in China is believed to have used a permanent magnet as a compass [16]. Thus, one may be dealing here with reconsideration of one of the oldest chapters of physics.

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