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Multi Foci Closed Curves

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Abstract: A complete mathematical framework for continuously differentiable curves having more than two non-collinear foci, discovered under this study, has been proposed in this paper. This would be an extension of Analytical Geometry as these curves are in a similar class to ellipses. They are appropriate engineering graphics and helpful in many sciences especially in nuclear, biosciences and astrophysics.
Keywords:: Euclidean Analytic Geometry, Classical geometry, Engineering graphics, Geometry Education.

1 INTRODUCTION:

There are basically two approaches to the only known continuously differentiable closed curve having more than one focus, i.e. ellipse, for its definition and equation. One by defining the directrix and a focus [1] and the other by using the sum of the focal radii [2]. The later one is proved to be the most suitable approach for the proposed concept of Multifoci Closed Curves. As yet, the concept of unifocus (centre) of a circle and bifoci of an ellipse have been defined by mathematicians. The concept of Multifoci (noncollinear foci) occurring after a big time-gap is brainchild of my late mother Aquila who initiated the idea of multi-foci later I developed it. I also call them as Aquila Curves. Only three, four, and five noncollinear foci have been analyzed under this study and the curves consequence of them are named as Triellipse, Quadraellipse, and Pentaellipse. Each curve can have a number of shapes, which depend upon the change in the placement of the foci. They are in the category of ellipses.

1.12 TRACING METHOD

Tracing is quite similar to that of ellipses. Refer Figure 1-A.. Draw three lines MO, KN, and RL in such a way that they form a triangle. Locate intersecting points. Foci A, B and C. Insert a pin at each focus point and tie a piece of thread in the form of a loop around the pins in such a way that the pencil point when placed in the loop (keeping the thread tight) is just at the end P_0 on the line MO. Move the pencil around the foci maintaining an even tension in the thread throughout and obtain Triellipse. The same method will be adopted to trace out the Quadraellipse and Pentaellipse.

1.13 GEOMETRICAL ANALYSIS OF TRACING

The paper introduces two terminology's viz. Relative foci and Relative focal distance.

1.14 SITUATION-1

While tracing any curve, the moving point becomes collinear with any two of the foci at any stage of movement. In such a tracing the focus, found on the first line of collinearity, nearest to the moving point and likewise the focus found on the second line of collinearity, nearest to the moving point are termed as 'Relative Foci' for every point on the curve from first line of collinearity to the second line of collinearity provided these foci are distinct.

In Figure 1-A the moving point P forms BCP_0 a line of collinearity and BAP_3 the next line of collinearity. Focus C and A are nearest to the moving point's position P_0 and P_3 respectively. They also fulfill the condition of their being distinct. Hence foci A and C will be 'Relative Foci' for every point on the curve form P_0 to P_3. The distance between the Relative Foci is called **Relative Focal Distance**. Foci C and A are relative foci. Hence CA will be the relative focal distance

Figure 1-A

Triellipse

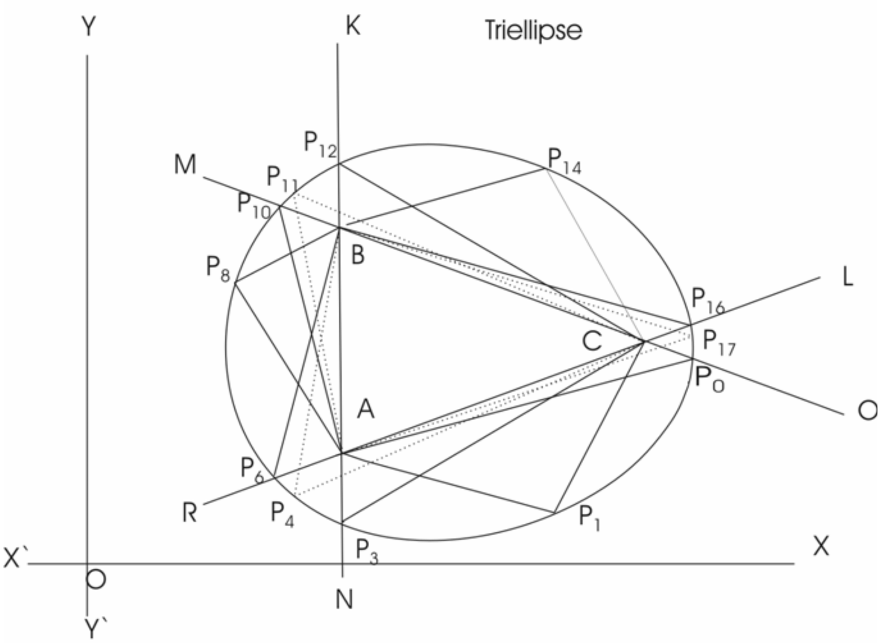
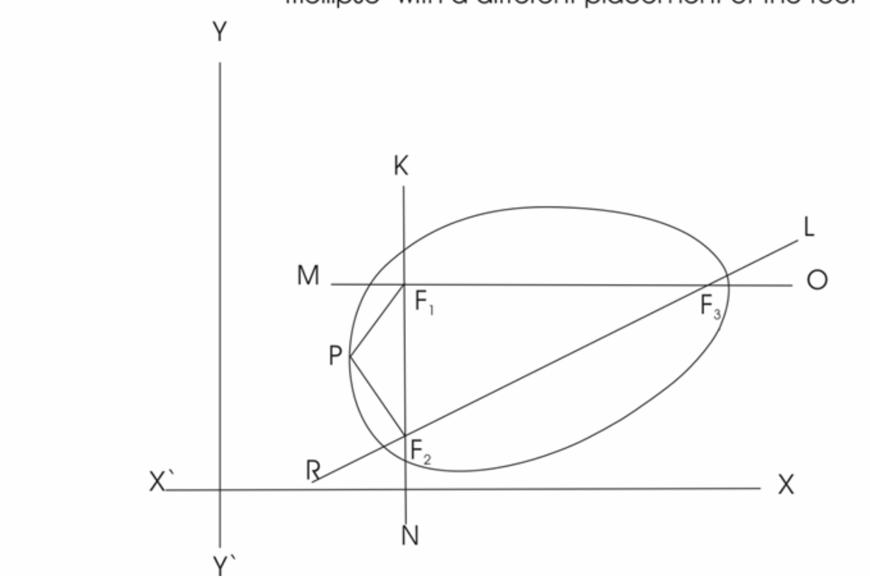


Fig 1-B

Triellipse with a different placement of the foci



1.15 SITUATION –2

In a situation where the two foci described above are common but not distinct, the other two foci (other than common) lying distinctly on first and second lines of collinearity will be 'Relative Foci' for every point on the curve from first line to the second line of collinearity.

P moves from P_3 to P_6 forming BAP_3 first and CAP_6 second lines of collinearity. Focus A alone is nearest to both P_3 and P_6 being common and not distinct, hence it is left out (skipped). Now B and C are two distinct focuses other than A on first and second line of collinearity respectively. Hence they will be Relative foci for every point from P_3 to P_6. In this case the **Relative Focal Distance** will be the sum of the distances of a common focus from the Relative Foci. Focus A is a common focus B and C are relative foci. Hence (AB + AC) will be the relative focal distance.

Between every pair of lines of collinearity, the curves are elliptic curves. The difference between the relative focal distance and the sum of relative focal radii is always constant.

For Example:

1 between lines BCP_0 and BAP_3 .

$|P_1 A| + |P_1 C| - |A C| = \lambda$ where $|A C|$ is Relative Focal Distance and A and C are Relative Foci.

2 Between BAP_3 and CAP_6

$|P_4 B| + |P_4 C| -(|A B| + |A C|) = \lambda$ where $(|A B| + |A C|)$ is Relative Focal Distance and B and C are Relative Foci.

Correspondingly between every such pair of lines of collinearity λ will be obtained.

1.3 DEFINITIONS SECTION

Triellipse is defined as a curve, with three noncollinear foci, traced out by a point moving in the same plane in such a way that the sum of its distances from relative foci bears a constant difference to the relative focal distance.

Ref. Fig 1-B : Let P be a point on Triellipse with F_1 and F_2 as Relative Foci. According to the definition of triellipse following relation is derived

$$|PF_1| + |PF_2| - |F_1F_2| = \lambda_t$$

Where PF_1 and PF_2 are the distances of the moving point P from the Relative Foci F_1 and F_2 respectively. F_1F_2 is the Relative Focal Distance and λ_t is the coplanar constant of triellipse and it signifies that though the foci are noncollinear yet they are in the same plane .

Now suppose the coordinates of moving point P , foci F_1 and F_2 are (x, y) , (h, k) and (l, m) respectively. The required equation of Triellipse will be:

$[(x - h)^2 + (y - k)^2]^{1/2} + [(x - l)^2 + (y - m)^2]^{1/2} - R = \lambda_t$ where $R = F_1F_2$ the Relative Focal Distance.

Quadraellipse is defined as a curve, with four foci three being noncollinear, traced out by a point moving in the same plane in such a way that the sum of its distances from relative foci bears a constant difference to the relative focal distance.

Ref. Fig. 2-A: Let P be a point on Qudraellipse with F_1 and F_3 as Relative Foci. According to the definition of Quadraellipse following relation is derived

$$|PF_1| + |PF_3| - |F_1F_3| = \lambda_q$$

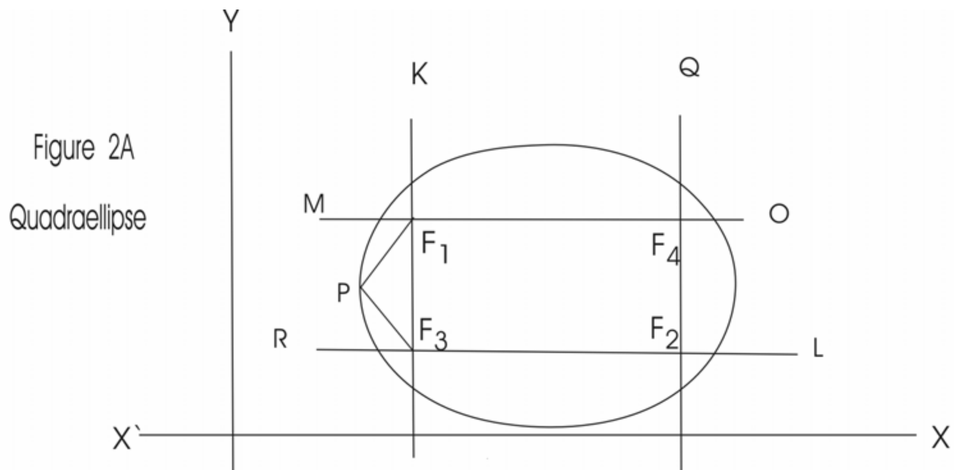
F_1F_3 is the Relative Focal Distance and λ_q is the coplanar constant of Quadraellipse.

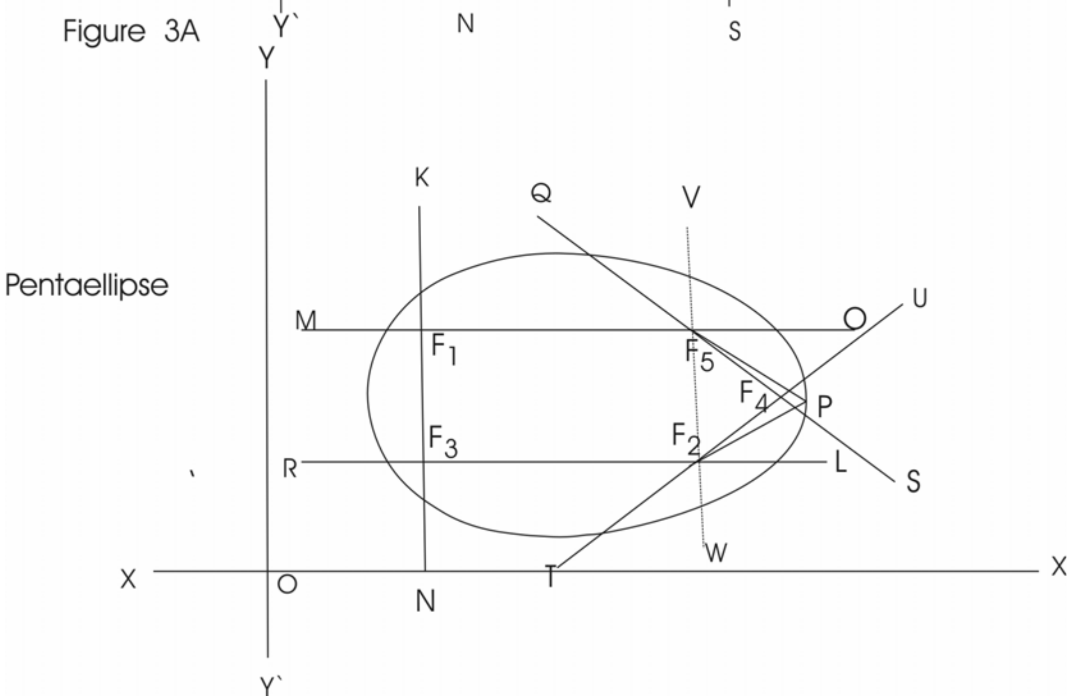
Pentaellipse is defined as a curve, with five foci three being non-collinear, traced out by a point moving in the same plane in such a way that the sum of its distances from relative foci bears a constant difference to the relative focal distance.

Ref. Fig. 3-A: According to the definition of Pentaellipse following relation is derived

$$|PF_2| + |PF_5| - (|F_4F_2| + |F_4F_2|) = \lambda_p$$

where P is a moving point, $F_1$and F_2 are Relative Foci. $(|F_4F_2|+|F_4F_2|)$ is the Relative Focal Distance & λ_p is the coplanar constant of Pentaellipse.





Note:

If three lines of collinearity meet at the same focus first and third lines are considered as the first and the second lines of collinearity. Middle one is skipped. Here VW is the middle line of collinearity hence it is left out.

Mechanical method of tracing justifies the continuity of the radius of the curvature whereas geometric analysis of the tracing justifies the differentiability.

1.4 APPLICATION AND RESULTS

The Multifoci Closed Curves will add new dimensions to Analytical Geometry (Coordinate Geometry) and other subjects related to mathematics beyond the prevailing concept of ellipse.

1.41 Nuclear Physics:

The Multifoci Closed Curves can be applied to nuclear sciences and result will be surprisingly remarkable. This theory suggests that the electron orbit will vary with the changes in the shape of the nucleus. It clubs the two great concepts, the concept of classical physics (1) an electron behaves as a particle and moves in standard orbits around the nucleus [3]; and (2) the concept of quantum mechanics which indicates the wave property of the electron and also the probability of the electron clouds [4].

J. Rain Water an American physicist, in 1950 suggested that the nuclear core consisting of even nucleons will have spheroidal shape rather than spherical one. Taking clue from this it can be analyzed that the nucleus consisting of three nucleons arranged in any order occupying the least possible space will have only triangular shape. Likewise the nucleus with four, five, six nucleons and so on occupying the least possible space will give the shapes accordingly. Only the large number of nucleons will jumble up in spherical or spheroidal shapes. Evidently the distribution of proton charge will be in accordance with the shape of the nucleus and maximum charge will be on the vertices, which behave as foci to determine the path of electron around them. If the shape of the nucleus is triangular, the electron chooses trielliptical orbit. Likewise if the shape of the nucleus is quadrangular the orbit of the electron is quadraelliptical and so on. In circular orbit electron possesses one degree of freedom because the centre is the only fixed point. In Somerfeld's elliptic orbit where foci are two distinct fixed points, electron possesses two degrees of freedom. Likewise in triellipse, quadraellipse and in pentaellipse electron will possess more than two degrees of freedom. Despite the fact that the Multifoci curves are continuously differentiable, they can also be divided into many parts on the basis of pairs of lines of collinearity. These pairs are twice the number of foci. For half of such pairs all the n foci are used to trace curve between each such pair of them because all the foci remain connected with the thread of the loop in this situation. Such curves are called Identical Curves and they represent a certain energy level. For other half of the pairs (n-1) foci are used to trace curve between each such pair of them because common focus remain unconnected with the loop. Such curves are called discrete curves. Since each time (n-1) combination has a different foci arrangement, each such curve represents a different energy level not only different to identical ones but also different to one another. Identical curves with higher energy level and discrete curves with lower energy level one after the other throughout the orbit will create the wave. All identical curves having the same energy level will exhibit the phenomenon of spectral lines of doublet, triplet and quadruplet depending upon their numbers in the orbit. Thus, there are (n+1) different curves in Multifoci orbit. Each curve represents a certain energy level or sub-shell whereas Multifoci orbit itself represents a principal energy level or a shell. In different sub-shells electron should have different velocities and obviously the different wavelengths. Thus a wave packet comprises a group of such waves and the orbit itself represents the guiding wave [5]. The

Multifoci curve can be defined as a group curve in which the electron moves with a group velocity in the orbit. Electron wave can be adjusted in wavelength to fit an integral number of times around the circumference of a Bohr's orbit of radius r [6] and each wave joins smoothly with the next [4]. These conditions can only be justified when electron adopts Multifoci curves. Multifoci curves introduce entirely a new concept of waves according to which a wave is not necessarily to have a physical appearance to look alike (ups and downs) but it consists of different sub energy levels and yet it follows the smooth orbital path. The different energy levels give the orbit stability. This is how it clubs the classical theory of standard orbit and the quantum theory of wave's dynamics. Further more it supports the quantum concept of clouds just as at exited state when electron receives extra energy from external field, the shape of the orbit undergoes a change. It is quite possible because each Multifoci curve has a number of shapes, which depend, upon the placement of the foci. These changes in the shapes of orbit correspond to the tremor changes in the shape of electron clouds when electron takes jump.

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