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The Gravity as a Longitudinal Field

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Abstract: The wave equation valid for massless particles may have two solutions: $\mathbf{A} = \mathbf{e}\Phi$ (transverse vector potential) and $\mathbf{\Phi} = \mathbf{n}\Phi$ (longitudinal vector potential), **n** being the unit vector along the direction of motion and **e** the unit vector perpendicular to it. The transverse potential **A** leads to Maxwell's equations for the electromagnetic field. The vector potential **Φ** describes a longitudinal field which may interact with rest mass particles. The interaction between two rest mass particles via the longitudinal field leads to an expression for the potential energy identical to Newton's potential energy for gravity. The connection between the gravitation constants is of the form $\chi = 8\pi G/c^2 = \xi^2$, where G and χ are the Newton and Einstein gravitation constants respectively and $\xi= (d(\gamma)/dt)/|\mathbf{G}|$, where $\gamma = (1-v^2/c^2)^{-1/2}$ is the Lorentz factor and **G** the longitudinal field. The postulate of the equivalence between the inertial and gravitation masses is not necessary inasmuch as only the inertial mass of the particles during particle-longitudinal field interaction is considered.
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1. Introduction

The general relativity theory has occurred as a natural extension of the special relativity considering the uniform accelerated motion. The inclusion of the accelerated rectilinear motion in the construction of the relativity theory has been the attempt made by Einstein to understand the gravity and its consequences for the space–time properties.

We present in the following, an attempt to describe the gravitation interaction as the interaction between a particle with finite rest mass and a particle moving with the speed of light characterized by a field derived from a longitudinal vector potential. As we shall see below, the interaction between the particles with rest mass via the longitudinal field leads to an expression for the potential energy identical with Newton's gravitation potential energy.

2. The longitudinal photon in the free space

Let the wave equation fulfilled by particles moving with the speed of light in empty space

$$\left[\mathit{\Delta} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right] \mathit{\Phi} = 0 \qquad (1)$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$.

For $\mathbf{A} = \mathbf{e}\mathit{\Phi}$ ($\mathbf{e}$ is a unit vector perpendicular to the direction of motion and $\mathbf{A}$ the transverse vector potential), the wave equation (1) transforms into the Maxwell equations for the propagation of the electromagnetic field in empty space.[1] A photon described by an electromagnetic field can be called a transverse photon, because the electromagnetic field is derived from the transverse potential **A**.

By analogy with the transverse vector potential $\mathbf{A} = \mathbf{e}\mathit{\Phi}$, let us now define the function $\mathbf{\Phi}$ as longitudinal vector potential

$$\mathbf{\Phi} = \mathbf{n}\mathit{\Phi} \quad , \qquad (2)$$

where **n** is the unit vector orientated along the direction of motion.

To characterize the field generated by the vector potential $\mathbf{\Phi}$ we must first see the properties of the function $\mathit{\Phi}$ involved in the equations (1) and (2). This function describing the space-time evolution of the particle depends on the time variable $t' = t - r/c$, where $r=(x^2+y^2+z^2)^{1/2}$ is the displacement towards the positive axes.[1] It can generally be written as

$$\mathit{\Phi}(Ct') = \mathit{\Phi}(C(t-\mathbf{n}.\mathbf{r}/c)) = \mathit{\Phi}(C(t-(x^2+y^2+z^2)^{1/2})/c) \quad , \qquad (3)$$

where C is a constant and **n** is the unit vector along the direction of motion.

Since the particle described by the equation (1) is materialized as fields propagating with the maximum allowed speed "c" with the period T, we may consider the period T as the natural unit of time for such a particle. The time t' may accordingly be expressed in time units T proper to the particle

$$t' = nT, \quad n \in N \qquad (4)$$

We can choose the constant C in equation (3), so that

$$Ct' = CnT = 2\pi n. \qquad (5)$$

In this case we have

$$C = 2\pi/T = 2\pi\nu = \omega \qquad (6)$$

According to the equations (6), (5) and (3), the function $\mathit{\Phi}$ has a $2\pi n$ periodicity. It may be of the form

$$\mathit{\Phi} = A.e^{-i2\pi n} = A.\exp(-i\,(\omega t - \mathbf{k}.\mathbf{r})) \quad , \qquad (7)$$

where $\mathbf{k} = \mathbf{n}\omega/c$ is the propagation vector and A the real part of $\mathit{\Phi}$. We can easily see from the equations (7) and (2) that

$$\mathbf{G} = -\mathrm{grad}\mathit{\Phi} = 1/c\ \partial\mathbf{\Phi}/\partial t. \quad (8)$$

We shall call $\mathbf{G}$ the longitudinal field derived from the scalar potential $\mathit{\Phi}$ and vector potential $\mathbf{\Phi}$ respectively.

The wave equation (1) may be written, as follows

$$\mathbf{n}\Delta\mathit{\Phi} - 1/c\ \partial/\partial t[1/c(\partial\mathbf{\Phi}/\partial t)] = 0,$$

or using the expression (8) and the formula $\Delta = \mathrm{div}(\mathrm{grad})$

$$1/c(\partial\mathbf{G}/\partial t) + \mathbf{n}\mathrm{div}\mathbf{G} = 0. \quad (9)$$

Let the identity

$$\mathrm{div}(\mathbf{n}.|\mathbf{G}|^2) = \mathrm{div}((\mathbf{n}.\mathbf{G})\mathbf{G}) = \mathbf{G}.\mathbf{n}\ \mathrm{div}\mathbf{G} + \mathbf{G}.\mathrm{grad}(\mathbf{n}.\mathbf{G}). \quad (10)$$

From the equation (8), we have

$$\mathrm{grad}(\mathbf{n}.\mathbf{G}) = 1/c.\partial/\partial t(\mathrm{grad}\mathit{\Phi}) = -1/c(\partial\mathbf{G}/\partial t). \quad (11)$$

From the equations (10), (9) and (11) we finally get

$$\partial W/\partial t + \mathrm{div}\mathbf{S} = 0, \quad (12)$$

where

$$W = |\mathbf{G}|^2 \quad (13)$$

and

$$\mathbf{S} = \mathbf{n}c|\mathbf{G}|^2 = \mathbf{n}cW. \quad (14)$$

The quantities W and **S** in the equations 12-14 represent the energy density (W) and the energy flux respectively, orientated along the direction of motion. The equation (12) can be interpreted as an equation of continuity for the longitudinal field energy. It may also be written as follows

$$dW/dt = d(|\mathbf{G}|^2)/dt = \partial(|\mathbf{G}|^2)/\partial t + \mathrm{div}\mathbf{S} = 0 \quad (15)$$

The energy carried by the longitudinal field **G** can be found from

$$\int Wr^2dr = \int|\mathbf{G}|^2r^2dr = mc^2, \quad (16)$$

where mc^2 is the energy of the particle without rest mass moving with the speed of light and described by the longitudinal field **G**. Such a particle can be called a longitudinal photon.

If the equation (9) is multiplied by 2c**G** and then compared with the expression (12), we get another equivalent formula for the flux of energy density

$$\mathrm{div}\mathbf{S} = 2c|\mathbf{G}|.\mathrm{div}(\mathbf{G}), \qquad (17)$$

where $|\mathbf{G}| = \mathbf{n}.\mathbf{G}$.

3. The particle-longitudinal field interaction

We try in the following to analyze the interaction between a particle with finite rest mass and a longitudinal photon described in the section 2. Let there be a particle with inertial mass $M=\gamma M_0$, where M_0 is the rest mass and γ the Lorentz factor. Let us now consider the system consisting of a longitudinal photon-particle as being "closed" in the whole space. In other words, only the interaction between the longitudinal photon and that particle with finite rest mass is considered. The total energy of such a "closed" system will be equal to

$$mc^2 + Mc^2 = \mathrm{constant}, \qquad (18)$$

where mc^2 is the energy of the photon (see equation 16).

If

$$d(mc^2)/dt = -d(Mc^2)/dt \neq 0, \qquad (19)$$

the particle and the longitudinal photon will interact each other. If the energy Mc^2 of the particle is considered as being given by

$$\int (\rho Mc^2) r^2 dr = Mc^2 \qquad (20)$$

where (ρMc^2) is the energy density and $\int \rho r^2 dr = 1$, using the equations (20) and (16), equation (19) can be written as follows

$$d(W)/dt = -d(\rho Mc^2)/dt = -\rho d(Mc^2)/dt \neq 0. \qquad (21)$$

Expression d(W)/dt in the equation (21) is just the equation (12) or (15).[2] Therefore, we can write

$$dW/dt = d(|\mathbf{G}|^2)/dt = \partial(|\mathbf{G}|^2)/\partial t + \mathrm{div}\mathbf{S} \neq 0, \qquad (22)$$

where $\partial(|\mathbf{G}|^2)/\partial t$ indicates the local time differentiation, i.e. the change of $|\mathbf{G}|^2$ per unit time at a fixed point in space and dW/dt shows the change of energy density $W = |\mathbf{G}|^2$ per unit time when we follow the matter in its motion.[2]

The right-hand term in the equation (21) can be written, as follows

$$d(Mc^2)/dt = cd(p_4)/dt = cF_4, \quad (23)$$

where $p_4 = Mc = M_0c\gamma$ and $F_4 = d(p_4)/dt$, where p_4 and F_4 being the momentum and the force respectively. From the equations (21), (22) and (23) we have

$$\partial(|\mathbf{G}|^2)/\partial t + \mathrm{div}\mathbf{S} = -\rho c F_4. \quad (24)$$

For a static field, i.e. $\partial(|\mathbf{G}|^2)/\partial t = 0$, from the equations (24) and (17) we get

$$\mathrm{div}\mathbf{G} = -1/2\rho g, \quad (25)$$

where

$$g = F_4/|\mathbf{G}| = (d(p_4)/dt)/|\mathbf{G}| = M_0c\xi \quad (26)$$

and

$$\xi = (d(\gamma)/dt)/|\mathbf{G}|, \quad (27)$$

where $\gamma = (1-v^2/c^2)^{-1/2}$ is the Lorentz factor. Note that g has the dimensions of a charge $[M^{1/2}L^{3/2}T^{-1}]$ and ξ $[L^{1/2}M^{-1/2}]$. If the longitudinal field **G** derives from a scalar potential $\mathbf{G} = -\mathrm{grad}\Phi$ (see the equation (8)), then the relationship (25) becomes

$$\Delta\Phi = 4\pi\rho(1/8\pi)g. \quad (28)$$

This is the well-known Poisson equation having the solution

$$\Phi(r) = -(1/8\pi)g/r, \quad (29)$$

which is the scalar potential of the field produced by the charge g of the particle. Note that the solution $\Phi(r)$ is valid for a static field described by the expression (25) or (28), assuming that $\partial(|\mathbf{G}|^2)/\partial t = 0$. Note also that the formula (29) has been obtained using the expression (21) which may be regarded as the net conversion rate of the field energy from/into the mechanical energy of the particle.

If now the longitudinal field produced by a particle of charge g interacts with another particle of charge g' (the "charges" are different, depending on the rest mass values, see the expression (26)), the field-particle interaction must be described by the same relationship (21). In other words, the interaction between two particles via the longitudinal field **G** may be considered as a process of emission and absorption of longitudinal photons between particles. If the two particles are in the same reference system having the same speed v and neglecting the relativistic effects, then

$$g = M_0c\xi \text{ and } g' = M_0'c\xi\ , \quad (30)$$

where M_0 and M_0' are the rest masses of the particles and ξ is given by the expression (27). According to the last two formulas (29) and (30), the potentials of the two interacting particles are given by

$$\Phi(r) = -(1/8\pi)g/r \quad \text{and} \quad \Phi'(r) = -(1/8\pi)g'/r \tag{31}$$

and the potential energy by

$$U(r) = g'\Phi(r) = g\Phi'(r) = -(1/8\pi)gg'/r, \tag{32}$$

"r" being the distance between the two particles. From the equations (32) and (30) we have

$$U(r) = -(c^2\xi^2/8\pi)M_0M_0'/r = -GM_0M_0'/r, \tag{33}$$

where

$$G = c^2\,\xi^2\,/8\pi. \tag{34}$$

The obtained expression (33) for the potential energy shows a striking similarity with the well-known Newton's potential energy of the particle in a gravitational field produced by another particle. Observe that the potential energy U(r) is negative, because the rest masses M_0 and M_0' as well as ξ given by formula (27) ($d\gamma/dt \geq 0$) are always positive. Moreover, the constant ξ given by the expression (27) must be universal, because it does not contain quantities specific to the particle. The constant G in (34) must also be universal. If it is identified with Newton's constant, one obtains $\xi = 1.366.10^{-13}\ m^{1/2}kg^{-1/2}$. The connection with Einstein's gravitation constant χ [3] can easily be made using the expression (34)

$$\chi = 8\pi G/c^2 = \xi^2. \tag{35}$$

Observe that the "gravitational" masses in the equation (33) are identical with the inertial masses considered in (18), the postulate of their equivalency not being necessary.

Because the field **G** (see equation 8) mediating the gravitational interaction propagates along only one direction, the gravity cannot be screened in contrast with the electromagnetic interaction mediated by the electromagnetic field situated in a plane perpendicular to the direction of motion.

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