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Interaction Space and General Relativity

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Abstract: Interaction space suggestion tells us that we can/should use different spaces (coordinate systems) to describe different interactions. There are several methods to reach the idea of some special spaces (interaction spaces) for interactions. By this study; you will see one of these and you will notice that it is originating from general relativity.

Keywords: General Relativity, Interaction Space, Space-Time, Minkowski Space, Metric, Einstein Equations, Line Element

Although interaction space concept was suggested as a result of similarities between classical mechanics and field theory [1], the starting point of this concept was not these similarities and if truth be told, the general relativity and its origin played a very important role on suggesting interaction space. In this paper, we will try to show how the interaction space concept can be extracted from basics of general relativity. Also we will be able to learn that interaction spaces are 4-dimensional and Euclidean.

It is known that if there is no gravitation [2] the line element (or invariant interval) of the space-time [3] (or Minkowski Space) is

$$d\tau^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 \quad (1)$$

or by letting $x^0 = ict$, $x^1 = x$, $x^2 = y$ and $x^3 = z$

$$d\tau^2 = dx^\mu dx^\mu = g_{\mu\nu} dx^\mu dx^\nu, \quad (2)$$

where $g_{\mu\nu} = \text{diag}(-1,1,1,1)$ and it is metric of the flat space-time. The existence of gravitation changes the line element into

$$d\tau^2 = \eta_{\mu\nu}(x^\alpha) dx^\mu dx^\nu. \quad (3)$$

This time metric is a function of the space-time for the most general case and we denote it by $\eta_{\mu\nu}(x^\alpha)$. If we write Eqs. (2) and (3) as

$$d\tau^2 = dx^\mu dx^\mu \quad \Leftrightarrow \quad \text{There is no gravitation (Flat space-time),}$$

$$d\tau^2 = \eta_{\mu\nu}(x^\alpha) dx^\mu dx^\nu \quad \Leftrightarrow \quad \text{There is gravitation (Curved space-time).}$$

The line element (or metric) of a certain space is an invariant and distinctive quantity. That is, each space has a different and unique line element. But by comparing line elements (2) and (3) we see that they are totally different. So, **two different line elements mean there are two different spaces**. Therefore; we can conclude that **in existence of gravitation, curved space-time is not the space-time (Minkowski space) anymore and there is a new space now** (That is, the curvature of space-time is resulted from this new space.). As you guess, this new space is *interaction space*.

If there is a space, then what are its properties? Is it Euclidean or not? How many dimensions does it have? We can find answers of all these questions by using only the line element (3). For this we define some matrices

$$H = \begin{pmatrix} \eta_{00} & \eta_{01} & \eta_{02} & \eta_{03} \\ \eta_{10} & \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{20} & \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{30} & \eta_{31} & \eta_{32} & \eta_{33} \end{pmatrix} \quad D = \begin{pmatrix} dx^0 dx^0 & dx^0 dx^1 & dx^0 dx^2 & dx^0 dx^3 \\ dx^1 dx^0 & dx^1 dx^1 & dx^1 dx^2 & dx^1 dx^3 \\ dx^2 dx^0 & dx^2 dx^1 & dx^2 dx^2 & dx^2 dx^3 \\ dx^3 dx^0 & dx^3 dx^1 & dx^3 dx^2 & dx^3 dx^3 \end{pmatrix}.$$

Using these matrices the line element (3) becomes

$$d\tau^2 = \text{tr}(HD), \quad (4)$$

where $\text{tr}(\)$ denotes the trace process of a matrix. We can rewrite Eq. (4) by using an identity matrix I as

$$d\tau^2 = \text{tr}(IHID).$$

Since identity matrix is

$$I = CC^{-1},$$

where C is any arbitrary matrix and C^{-1} is inverse of C. So

$$d\tau^2 = \text{tr}(CC^{-1}HCC^{-1}D)$$

and since $\text{tr}(AB) = \text{tr}(BA)$

$$d\tau^2 = \text{tr}(C^{-1}HCC^{-1}DC) = \text{tr}[(C^{-1}HC)(C^{-1}DC)]. \quad (5)$$

Because C is an arbitrary matrix; we can choose it so that it diagonalizes H. That is

$$C^{-1}HC = \begin{pmatrix} h_{00} & 0 & 0 & 0 \\ 0 & h_{11} & 0 & 0 \\ 0 & 0 & h_{22} & 0 \\ 0 & 0 & 0 & h_{33} \end{pmatrix}.$$

We can write C^{-1} and C explicitly as

$$C = \begin{pmatrix} c_{00} & c_{01} & c_{02} & c_{03} \\ c_{10} & c_{11} & c_{12} & c_{13} \\ c_{20} & c_{21} & c_{22} & c_{23} \\ c_{30} & c_{31} & c_{32} & c_{33} \end{pmatrix} \quad C^{-1} = \begin{pmatrix} b_{00} & b_{01} & b_{02} & b_{03} \\ b_{10} & b_{11} & b_{12} & b_{13} \\ b_{20} & b_{21} & b_{22} & b_{23} \\ b_{30} & b_{31} & b_{32} & b_{33} \end{pmatrix},$$

where $c_{\mu\nu}$ and $b_{\mu\nu}$ elements of C and C^{-1} and they are some functions of space-time. Thus

$$C^{-1}DC = \begin{pmatrix} b_{0,\mu} dx^\mu c_{\nu 0} dx^\nu & b_{0,\mu} dx^\mu c_{\nu 1} dx^\nu & b_{0,\mu} dx^\mu c_{\nu 2} dx^\nu & b_{0,\mu} dx^\mu c_{\nu 3} dx^\nu \\ b_{1,\mu} dx^\mu c_{\nu 0} dx^\nu & b_{1,\mu} dx^\mu c_{\nu 1} dx^\nu & b_{1,\mu} dx^\mu c_{\nu 2} dx^\nu & b_{1,\mu} dx^\mu c_{\nu 3} dx^\nu \\ b_{2,\mu} dx^\mu c_{\nu 0} dx^\nu & b_{2,\mu} dx^\mu c_{\nu 1} dx^\nu & b_{2,\mu} dx^\mu c_{\nu 2} dx^\nu & b_{2,\mu} dx^\mu c_{\nu 3} dx^\nu \\ b_{3,\mu} dx^\mu c_{\nu 0} dx^\nu & b_{3,\mu} dx^\mu c_{\nu 1} dx^\nu & b_{3,\mu} dx^\mu c_{\nu 2} dx^\nu & b_{3,\mu} dx^\mu c_{\nu 3} dx^\nu \end{pmatrix}$$

can be obtained. Let $c_{\mu\nu} = \frac{\partial u_\mu}{\partial x^\nu} = \partial_\nu u_\mu$ and $b_{\mu\nu} = \frac{\partial y_\mu}{\partial x^\nu} = \partial_\nu y_\mu$. Then

$$\frac{\partial_\nu u_\mu}{\partial_\nu y_\mu} = \frac{c_{\mu\nu}}{b_{\mu\nu}} \quad (6)$$

and

$$C^{-1}DC = \begin{pmatrix} dy_0 du_0 & dy_0 du_1 & dy_0 du_2 & dy_0 du_3 \\ dy_1 du_0 & dy_1 du_1 & dy_1 du_2 & dy_1 du_3 \\ dy_2 du_0 & dy_2 du_1 & dy_2 du_2 & dy_2 du_3 \\ dy_3 du_0 & dy_3 du_1 & dy_3 du_2 & dy_3 du_3 \end{pmatrix}$$

can be written. Thus Eq. (5) becomes

$$d\tau^2 = h_{00} dy_0 du_0 + h_{11} dy_1 du_1 + h_{22} dy_2 du_2 + h_{33} dy_3 du_3.$$

From Eq. (6)

$$du_\mu = \frac{\partial u_\mu}{\partial x^\nu} dx^\nu = \frac{\partial u_\mu}{\partial x^\nu} \frac{\partial x^\nu}{\partial y_\mu} dy_\mu = \frac{\partial_\nu u_\mu}{\partial_\nu y_\mu} dy_\mu = \frac{c_{\mu\nu}}{b_{\mu\nu}} dy_\mu. \quad (7)$$

Let

$$l_\mu = \frac{c_{\mu\nu}}{b_{\mu\nu}}. \quad (8)$$

In Eqs. (7) and (8), there is summation over ν but no over μ . So

$$d\tau^2 = h_{00} l_0 dy_0 dy_0 + h_{11} l_1 dy_1 dy_1 + h_{22} l_2 dy_2 dy_2 + h_{33} l_3 dy_3 dy_3. \quad (9)$$

The line element (9) indicates there is 4-dimensional Euclidean space. We can write it in a different form by letting $\sqrt{h_{00}l_0} dy_0 = dT_0$, $\sqrt{h_{11}l_1} dy_1 = dT_1$, $\sqrt{h_{22}l_2} dy_2 = dT_2$, $\sqrt{h_{33}l_3} dy_3 = dT_3$ as

$$d\tau^2 = dT^\mu dT_\mu, \quad (10)$$

where dT^μ is complex conjugate of dT_μ ($T^\mu = (T_\mu)^*$) since dT_μ can be complex. If we define a four-dimensional vector

$$\mathbf{R} = T_\mu \mathbf{A}_\mu, \quad (11)$$

where \mathbf{A}_μ are some unit vectors and they satisfies

$$\mathbf{A}_\mu \cdot \mathbf{A}_\nu = \delta_{\mu\nu},$$

the line element (10) becomes

$$d\tau^2 = \eta_{\mu\nu}(x^\alpha) dx^\mu dx^\nu \equiv d\mathbf{R}^* \cdot d\mathbf{R}, \quad (12)$$

where \mathbf{R}^* is complex conjugate of \mathbf{R} . Since $d\tau^2$ is a line element; \mathbf{R} must be position vector of any point in a new four-dimensional Euclidean space. So we have seen that the line element (3) (or curved space-time) which has arisen due to gravitation established a four-dimensional Euclidean space. Similar to other Euclidean spaces there must be some coordinate axes and unit vectors. Using these we can reach some interesting results about gravitation as we did for electromagnetism [1].

Then presence of gravitation involves a new 4-dimensional Euclidean space or coordinate system. However we suppose that not only presence of gravitation but also presence of electromagnetism and other interactions involve new spaces. Briefly we expect each type interaction involves different type space. Or inversely; different types of spaces may describe different types of interactions. That is why we call these spaces as **interaction space**.

References

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