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Energy-based Model to Contract the Riemann Curvature Tensor

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Abstract: In this article it is proposed that the geometry of space could be defined solely based on energy preservation principle. Energy based model is almost metric theory. However, as the speed of an object increases, its behavior changes from space-like to more time-like. Though the results are very close to those defined by General Theory of Relativity (GTR), there are several benefits compared to the GTR. For example, the equations are much simpler (especially force and energy) and the space defined is continuous. The new model gives new tools to build up a "global" theory. At the end of the article, some tests to check which of the models give correct results are proposed.

Keywords: Gravitation, General Relativity.

1. Introduction

The necessary condition for a space to have a Lorentz metric is as $R^{\alpha}{}_{\eta\beta\gamma} = 0$, where $R^{\alpha}{}_{\eta\beta\gamma}$ is the Riemann curvature tensor. Let φ be gravitational potential. According to GTR, it is supposed that metric tensor is of form $g_{00} = 1 + 2\varphi / c^2$ like it is for slowly moving object in a gravitational field according to Newton's second law. Then,

$$\sum_{i=1}^{3} g_{00|i|i} = \sum_{i=1}^{3} \varphi_{|i|i} \equiv 0, \tag{1}$$

and the only meaningful contraction is as $R_{\eta\gamma} = 0$. What if g_{00} is not exactly $1 + 2\varphi / c^2$? Evidently, a small change in local gravitational model can drastically change global behavior of gravitational field (i.e., the approach to define metric tensor based on contracting Riemann tensor is very sensitive in respect to the selected gravitational model). There exists many different models which approximately solve the Riemann curvature tensor simultaneously as they very precisely give same results as the classical gravitational model under reasonable conditions. However, each of these models may define very different physical behavior in extreme conditions. How to define a consistent contraction of the Riemann conditions?

The speed of a freely falling object at general relativistic Schwarzschild field measured by a distant observer is as (Zeldovich, Novikov: Relativistic Astrophysics. Volume 1)

$$v = \frac{dr}{dt} = c(1 - r_g / r)\sqrt{1 - \frac{1 - r_g / r}{1 - r_g / r_0}} \tag{3}$$

where $r_g = 2GM/c^2$. Suppose $r_0 = \infty$. The total energy gets form

$$E_K = \frac{m_0 c_L^2}{\sqrt{1-(v/c_L)^2}} - m_0 c_L^2 = m_0 c_L^2 \left(\frac{1}{\sqrt{1-r_g/r}} - 1\right), \quad (4)$$

where c_L is speed of light at distance r. Thus, total energy must be corrected by term $\sqrt{1-r_g/r}$.

In this article such an approach to contract Riemann tensor is presented, that the total energy doesn't need extra correcting terms. While static case is considered, energy based model can also be described by its metric properties, and the curvature tensor coefficients are very close to those defined by GTR. An extra term is needed in dynamic case, to both achieve a consistent energy formula as well as to support infinity (to keep the overall force balance within the Universe). The extra term describes how a space-like behavior changes towards time-like behavior as the speed increases.

The basic idea behind the energy-based model is simple: Suppose an object is dropped. In the beginning of drop the object has mainly potential energy. The potential energy is related to the gravitational force earth pulls on the object. Gradually, while the speed is increased, the kinetic energy takes more dominant role over the potential energy. At the end the object hits the floor, and a portion of the kinetic energy is changed to temperature and radiated. Thus the total energy of the system decreases. Thus also earth's ability to have potential energy must also decrease, and if a similar object as earlier is dropped, it will not accelerate as rapidly as the first object. The difference is extremely small, but it exists and has to be taken into account if high accuracy is required. The new model states that:

- As two or more objects interact, they have certain interaction energy.
- All forces are based on interaction energy.
- The interacting energy of objects is decreased as much as the amount of work done by the interaction.

2. Static Model

All work done is due to the interaction energy. The interaction energy is decreased by the amount of the work done due to the interaction energy. If U_1 is the interaction energy of one object and U_2 is the interaction energy of an other object, the objects interface with each other with force $F(r) = \alpha(r) U_1 U_2$. The work done by this force is $W = \int F(r)dr = \int \alpha(r) U_1 U_2 dr$ for both of the objects. The interaction energy of object one with initial interaction energy of U_1 is decreased to $U = U_1 - W$. Thus actually interaction energy has same units as work and interaction energy is also a function distance. The interaction energy related to a mass m is $U_m = mv_L^2$, where v_L is defined to be the limit velocity for the object. The limit velocity represents the velocity, which the object can ultimately achieve without being split into smaller pieces. Interaction energy is actually that portion of the energy of the object, which causes interaction between other objects. The work done during interaction between objects is normally so small, that one cannot measure the change in interaction energy.

According to the definition of interaction energy

$$F_{m,r} = -\frac{\partial U_m}{\partial r} \quad . \quad (5)$$

For a central force

$$F_{m,r} = -\frac{kU_m U_M}{r^2} \quad , \tag{6}$$

because

$$\frac{\partial U_{m,r}}{\partial r} = \frac{\partial U_{M,r}}{\partial r}\,, \tag{7}$$

$$U_{M,r} = U_{M,\infty} - (U_{m,\infty} - U_{m,r})\,, \tag{8}$$

thus

$$\frac{\partial U_{m,r}}{\partial r} = \frac{kU_{m,r}U_{M,r}}{r^2} = \frac{kU_{m,r}(U_{M,\infty} - U_{m,\infty} + U_{m,r})}{r^2} \quad . \tag{9}$$

Using $\alpha = U_{M,\infty} - U_{m,\infty}$ and $u = U_{m,r}$, the equation 9 get form

$$\frac{du}{dr} = \frac{ku(\alpha + u)}{r^2}\,. \tag{10}$$

If M>>m, the solution is

$$U_{m,r} = U_{m,\infty} e^{-\frac{kU_M}{r}} \quad . \tag{11}$$

Taking Taylor series expansion and substituting it to equation 6 gives the same result as classical gravitational theory (with accuracy to fourth order terms in distance). When distance is reasonable, the difference cannot be measured.

The limit velocity at distance r is

$$v_{L,r} = \sqrt{\frac{U_{m,r}}{m}} = \frac{\sqrt{U_{m,\infty}}}{\sqrt{m}} e^{-\frac{kU_M}{2r}} = v_{L,\infty} e^{-\frac{kU_M}{2r}}\,. \tag{12}$$

If we define $c = v_{L,\infty}$, we get

$$v_{L,r} = ce^{-\frac{kU_M}{2r}}\,. \tag{13}$$

Suppose $v = v_L$. Acceleration gets form

$$a_r = v_{L,\infty} e^{-\frac{kU_M}{2r}} \left(\frac{kU_M}{2r^2}\right)\frac{dr}{dt} = \frac{kU_M}{2r^2} v_L v = \frac{kU_M}{2r^2} v_L^2\,. \tag{14}$$

Related force F is thus

$$F = ma_r = \frac{kU_M}{2r^2} m v_L^2 = \frac{1}{2}\frac{kU_M U_m}{r^2} = \frac{1}{2}\frac{\partial U}{\partial r}\,. \tag{15}$$

The symmetry suggests that in dynamic case, when objects are moving respect each other, the pushing force would in the limit velocity be same as the pulling force when the objects are in rest. Thus, the force formulae should be corrected by velocity dependent part, and the general formulae for interaction energy should have the form

$$\vec{F}_m(r,v) = -\frac{\partial U_m}{\partial r}\vec{u}_r + \frac{3}{2}\left(\frac{v}{v_{L,r}}\right)^2 \frac{\partial U_m}{\partial r}\vec{u}_v \tag{16}$$

supposed the objects are moving in opposite directions.

2.1 Metric tensor related to interaction energy

If $M >> m$ the coefficients of the metric tensor are as

$$g_{11} = e^{-kM/r}; g_{22} = -\frac{\mu e^{kM/r}}{v_{L,\infty}^2}; g_{33} = -r^2/v_{L,\infty}^2; g_{44} = -r^2 \sin^2\theta / v_{L,\infty}^2 \ . \tag{17}$$

With reasonable large r the difference between GTR and energy-based model is very small. For example the Taylor expansion of limit velocity is as

$$v_{L,r} = c\left(1 - \frac{kU_M}{2r} + \frac{(kU_M)^2}{2!(2r)^2} - ...\right) \ . \tag{18}$$

The limit velocity is thus same as speed of the light close to a massive body defined by GTR until second order coefficients.

Similarly as for GTR, all the Christoffel symbols are not null and thus neither of the solutions solve all the equations of $R^{\alpha}{}_{\eta\alpha\gamma} = 0$. For example, R^1_{010} is as

$$R^1_{010} = \frac{kU_M}{r^2} {v_{L,\infty}}^2 e^{-\frac{2kU_M}{r}} \left(\frac{2}{r} - \frac{kU_M}{r^2}\right) \ . \tag{19}$$

Thus Riemann curvature tensor is not identically zero but it tends rapidly to zero as r increases. Surprisingly, Riemann tensor coefficients tend faster to zero than those defined by GTR !

3. Dynamic model

It will now be shown that if the general formulae for force is defined to be

$$\vec{F}_m(r,v) = -\frac{\partial U_m}{\partial r}\vec{u}_r + \frac{3}{2}\left(\frac{\vec{v}\cdot\vec{u}_r}{v_{L,r}}\right)^2 \frac{\partial U_m}{\partial r}\vec{u}_v , \tag{20}$$

then no “relativistic” correction for total energy is needed.

Suppose object with mass m is moving directly towards the object of mass M (M>>m). The force is thus

$$F_m(r,v) = -\frac{\partial U_m}{\partial r} + \frac{3}{2}\left(v/v_{L,r}\right)^2 \frac{\partial U_m}{\partial r} . \tag{21}$$

Using equations for interaction energy and velocity, this can be led to form

$$dv/dt = \frac{kU_M}{r^2} v_{L,\infty}{}^2 e^{-kU_M/r} \left(\frac{3v^2 e^{kU_M/r}}{2v_{L,\infty}{}^2} - 1\right) \quad . \tag{22}$$

Solving this and supposing that the fall begins at infinity gives

$$v = dr/dt = v_{L,r}\sqrt{1 - e^{-2(kU_M/r)}} = v_{L,r}\sqrt{1 - \frac{v_{L,r}^4}{v_{L,\infty}^4}} \quad . \tag{23}$$

Define kinetic energy for a freely falling object is thus as

$$E_K = \frac{m_0 v_L^2}{\sqrt{1-(v/v_L)^2}} - m_0 v_L^2 \quad . \tag{24}$$

Substituting $v = v_{L,r}\sqrt{1-(\frac{v_{L,r}}{v_{L,\infty}})^4}$ gives

$$E_K = m_0(v_{L,\infty}^2 - v_L^2) , \tag{25}$$

i.e. the change of kinetic energy is exactly the same as the change of interaction energy. The result is remarkable in the sense that according to a local observer in rest the measured mass of a freely falling object increases exactly as expected according to the STR. However, according to a distant observer the total energy is not changed.

4. Conclusions

The General Theory of Relativity explains that interactions between masses are due to curving of space and time. The key differences between the two approaches are listed in Table 1. Based on these differences it shouldn't be difficult to select which of the models one should concentrate on.

Is it possible to find any measurable difference between GTR and the energy-based model? Some tests that may be possible to implement are suggested:

1. Vibration of two interacting rapidly rating bars should differ from that predicted by GTR (Figure 1)
2. Interaction energy drops (Figure 2)
3. the detailed spectra of energy bursts generated by drops of interaction energy.

The drop in interaction energy can be huge from the surface to the center of a star. Thus the difference of interaction energy can be one of sources of radiation energy within stars, though the decrease of interaction energy might also work as a catalyst for fusion by decreasing electromagnetic forces compared to the masses. A rapid decrease within interaction energy might be one source for Nova energy formation. It can explain the speed how rapidly a nova can flame up.

Also, energy-based model can give simple explanation to rapid oscillation in radiation energy, because oscillating and radiating volume can be small, though the total volume could be large. When an object is dropping towards a massive object, all material it hits causes decrease in total energy and radiation is generated. Actually, this phenomenon has been also kept as a proof of existence of black holes. However, the denser the object is, the larger the difference in behavior.

Also, based on energy-based model, the entire center volume is radiating energy, not only spherically shaped volume outside Schwarzschild radius. The difference in radiation distribution may be measurable.

Table 1. Key Differences between GTR and Energy-based model.

	Topic	**GTR**	**Energy-based model**
1	Reasoning	Based on guess that gravitational field should resemble classical theory	Based on conservation of the total energy
2	Simplicity	Requires 16 equations. Especially energy formulae need correction.	Can be described by one equation. However, the solution of n-body objects is tedious, though possible with numerical methods. Simple energy formulae
3	Quality of the contracted Riemann tensor	Riemann tensor coefficients tend to zero as r increases	Riemann tensor coefficients tend faster to zero than according to GTR
4	Black Holes	Relativistic singularity	No Block Holes exist
5	Support to Quantum mechanics	No simple connection found so far	May explain quantification, uncertainty etc. because the two components of force can tune what so ever type of force field, and because there exist limit for a maximum of interaction energy density.
6	Infinity of Universe	No support	Supports

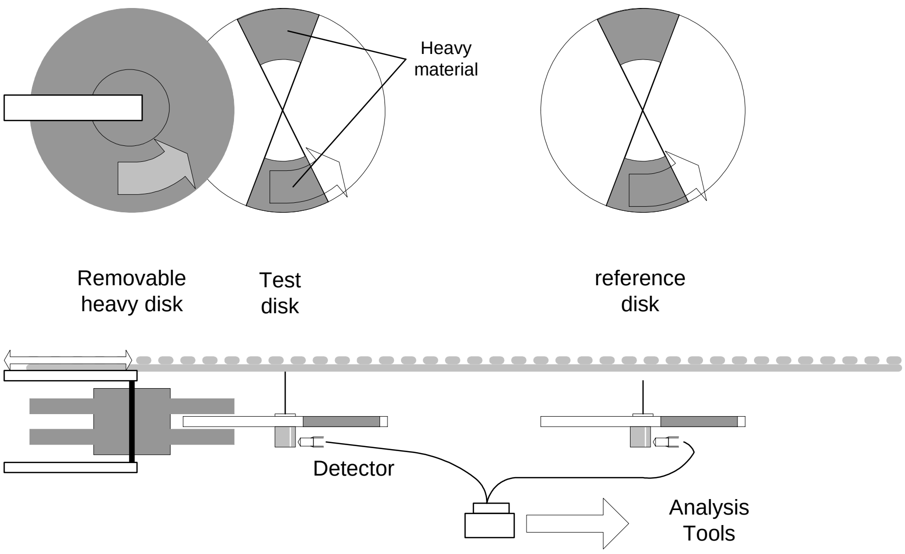

Figure 1. Experimental arrangement to test gravitational push. When the removable rapidly rotating disk-system is brought close to the test disk, the rotating speed decreases as either of the heavy material bars of the test disk are between removable disks. The rotating speed of the test system decreases faster than the rotating speed of reference disk, and the difference might be measurable.

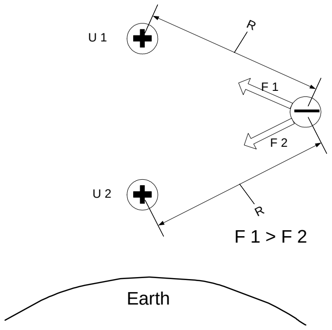

Figure 2. Because all forces are due to interaction energy, it might be possible to measure the change in the electromagnetic force between two charged objects when the object is moving towards earth. The electronic force between two objects should decrease as object drops in gravitational field, i.e if U2 < U1 then F2 < F1 also in case of electromagnetic forces.

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