Journal of Theoretics
Volume 6-1, Feb-March 2004

The Quarks as Dirac States

Costinel.I.Lepadatu costinellro@yahoo.com
Institute of Chemical Physics, Spl.Independentei 202,
Bucharest 77208, Romania

Abstract: As fermions, the quarks and leptons of the first family could be described as pairs of solutions of the Dirac equation (Dirac states). The color and electric charges are associated with the symmetry operations bringing the Dirac states into pairs. The obtained quark model is similar to that given by Han – Nambu, where the electric charges of the d and u quarks assume integer values. The quark s may have an inner structure formed from quarks of the first family, $s = u^*ud$ and $s^* = uu^*d$.

Keywords: Quarks, Dirac states, Han-Nambu model, s quark.

The quarks as fundamental constituents of matter are considered physical objects gifted with two spins: S_3 (external) and I_3 (internal). As Fermi-like particles, they must obey the Dirac equation of motion. For a spinning particle in its rest frame, the Dirac equation offers four solutions describing the following physical states:

1. $E_1 = +m_0c^2$; $S_z = +\eta/2$. 2. $E_2 = +m_0c^2$; $S_z = -\eta/2$.
3. $E_3 = -m_0c^2$; $S_z = -\eta/2$. 4. $E_4 = -m_0c^2$; $S_z = +\eta/2$,

where m_0 is the rest mass of the particle and S_z the "z" component of the spin momentum.

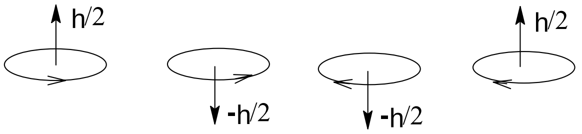
These four Dirac solutions must fulfill the fundamental quantum relationship for the wave - particle duality:

$$\pm m_0c^2 = \pm \omega_0 \eta \quad , \tag{1}$$

where the left - hand side that is the relativistic energy of the spinning particle in its rest frame describes the corpuscular behavior of the particle and the right - hand side the wavelike one.

If now, ω_0 in equation (1) is considered as the spin angular velocity [1] of the particle, then the right - hand side of this equation may be regarded as another relativistic expression for the rest energy. This assumption seems to be correct, because the invariance of the relativistic space-time interval $ds^2 = c^2dt^2 - dr^2$ to such a spin motion ω_0 of the particle (spin isotropy) leads to the angular (spin) momentum $S_z = m_0\,|(x.dy/dt-y.dx/dt)| = \eta/2$, for all structureless elementary particles irrespective of their rest mass values.[2,3]

If this is so, then the signs of the mass values $\pm m_0$ occurring in the Dirac solutions might be related to the ± rotation directions of ω_0 (spinning). The Dirac states may schematically be represented as in Figure 1.



1. $\omega_0 > 0$ 2. $\omega_0 > 0$ 3. $\omega_0 < 0$ 4. $\omega_0 < 0$

Figure 1. The Dirac states for a spinning particle in its rest frame.

If the Dirac equation is now applied to the electron, the pair of Dirac states (1, 2) describes the electron, but the other pair (3, 4) describes its corresponding antiparticle, the positron. According to equation (1), both particles have the same positive mass value but different spinning orientations. Therefore, the Dirac equation is actually a single-particle equation describing a particle/antiparticle system. This particle system can be called Dirac particle, the four Dirac states being the possible physical hypostases of this spinning particle in the relativistic space-time frame.

Since e/e^+ particle system is represented as pairs of Dirac states, *the quarks as fermions must also be described as pairs of Dirac states, where the Dirac states are differently combined.* This assumption is strongly suggested by those two spins of the quarks, S_3 (external) and I_3 (internal).

We need eight Dirac particles, i.e. eight sets of Dirac solutions (8×4 = 32 Dirac states) as represented in Figure 1, capable to produce all possible 16 distinct pairs of Dirac states, by means of which the lepton/antilepton and quark/antiquark systems (i.e. e/e^+, υ/υ^*, d/d^*, u/u^*) could in principle be described. The arrangement of these 16 distinct pairs of Dirac states is presented in Table 1, where the pairs of spinning orientations marked with + and - are called colors (anticolor = color*).

According to the usual interpretation of the Dirac solutions, the electron and the positron are described by the Dirac pairs (1, 2) and (3, 4) respectively (Figure 1 and Table 1). Because these particles carry electric charges, these charges seem to exist only for those pairs, where the Dirac states have opposite spins but the same spinning sense (see the pairs (1,2) and (3,4) in Figure 1). The conventional sign of the electric charge is given by the spin of the second state from the pair and namely the state 2 for electron and the state 4 for positron.

By extension, one may assume that only those pairs of Dirac states carry an electric charge if their spins S_3, I_3 are opposite but the corresponding Dirac states have the same spinning orientation or in those cases, where S_3, I_3 are parallel but the Dirac states have opposite spinnings. The sign of the internal spin I_3 gives the sign of the electric charge.

Based on these assumptions, *the leptons e, ν and the quarks d ($S_3=+\eta/2$; $I_3=-\eta/2$) and u ($S_3=+\eta/2$; $I_3=+\eta/2$) as well as their antiparticles e^+, υ^*, d^*, u^* may be represented as distinct pairs of Dirac states* (Table 1). Their electric charges are given in parentheses, according to the previously assumed rules.

Table 1. The first family of quarks and leptons as distinct pairs of Dirac states.

Leptons:	Color(++)	Quarks:	Color(−−)	Color(+−)	Color(−+)
Antileptons	Color*(−−)	Antiquarks:	Color*(++)	Color*(−+)	Color*(+−)
e	1,2(−e)	$d(S_3=+\eta/2;I_3=-\eta/2)$	4,3(−e)	1,3(0)	4,2(0)
e^+	3,4(+e)	$d^*(S_3=-\eta/2;I_3=+\eta/2)$	2,1(+e)	3,1(0)	2,4(0)
υ	1,1(0)	$u(S_3=+\eta/2;I_3=+\eta/2)$	4,4(0)	1,4(+e)	4,1(+e)
υ^*	3,3(0)	$u^*(S_3=-\eta/2;I_3=-\eta/2)$	2,2(0)	3,2(−e)	2,3(−e)

As may be seen in Table 1, each row of three distinct pairs of Dirac states describe three quarks having the same S_3 and I_3 values. They strikingly remind the three colored quarks from Chromodynamics. The fact is interesting, because the colors could be connected with the symmetry operations bringing the Dirac states into pairs. The spinning orientations of the Dirac states seem to play a role in this context (they are denoted with + and –).

According to the rules assumed for the electric charge, only one d/d^* and two u/u^* quarks possess elementary electric charges. The quark model represented in Table 1 is very similar to that given, years ago, by Han-Nambu.[4] If the colors are not considered, the average electric charge of the quarks should be $q_d=-e/3$ and $q_u=2e/3$ as in the Gell-Mann standard model.

As regards the quarks belonging to the succeeding families of quarks, there are in principle three alternatives: they are built like *d* and *u* quarks from pairs of Dirac states; they are built from *d* and *u* quarks or from other constituents. Let us see the following decays[5-7], where the *s* quark from the second family of quarks is involved:

$$\begin{array}{ccccc} \Omega^- & \to & \Xi^0 & + & \pi^- \\ ss(s) & & ss(u) & & (du^*) \end{array} \tag{1}$$

$$\begin{array}{ccccc} \Omega^- & \to & \Xi^- & + & \pi^0 \\ ss(s) & & ss(d) & & (uu^*) \end{array} \tag{2}$$

$$\begin{array}{ccccc} \Omega^- & \to & \Lambda & + & K^- \\ ss(s) & & s(du) & & s(u^*) \end{array} \tag{3}$$

$$\begin{array}{ccccc} \Xi^- & \to & \Lambda & + & \pi^- \\ ds(s) & & ds(u) & & (du^*) \end{array} \tag{4}$$

$$\begin{array}{ccccc} \Xi^0 & \to & \Lambda & + & \pi^0 \\ us(s) & & us(d) & & (uu^*) \end{array} \tag{5}$$

$$\begin{array}{ccccc} \Sigma^- & \to & n & + & \pi^- \\ dd(s) & & dd(u) & & (du^*) \end{array} \tag{6}$$

$$\begin{array}{ccccc} \Lambda & \to & p & + & \pi^- \\ ud(s) & & ud(u) & & (du^*) \end{array} \tag{7}$$

$$\begin{array}{ccccc} \Lambda & \to & n & + & \pi^0 \\ ud(s) & & ud(d) & & (uu^*) \end{array} \tag{8}$$

$$\begin{array}{ccccc} \Sigma^+ & \to & p & + & \pi^0 \\ uu(s) & & uu(d) & & (uu^*) \end{array} \tag{9}$$

$$\begin{array}{ccccc} K^+ & \to & \pi^0 & + & \pi^+ \\ u(s^*) & & (uu^*) & & (d^*)u \end{array} \tag{10}$$

These decays suggest the structures u^*ud and uu^*d^* for the quarks *s* and s^*, irrespective of the mechanism assumed for these decomposition reactions (the decomposing *s* and s^* quarks are enclosed in brackets). The structures $s = u^*ud$ and $s^* = uu^*d^*$ result also from those decays, where both s and s^* quarks are decomposed:

$$\begin{array}{ccccccc} \eta' & \to & \eta & + & \pi^0 & + & \pi^0 \\ ss^* & & dd^* & & uu^* & & uu^* \end{array} \qquad (11)$$

$$\begin{array}{cccccccc} \varphi^0 & \to & \pi^+ & + & \pi^- & + & \pi^0 & . \\ ss^* & & ud^* & & u^*d & & uu^* & \end{array} \qquad (12)$$

Moreover, the following decays

$$\begin{array}{ccccc} \pi^+ & \to & e^+ & + & \upsilon_e \\ ud^* & & d^* & & u \end{array} \qquad (13)$$

$$\begin{array}{ccccc} \pi^- & \to & e & + & \upsilon_e^* \\ u^*d & & d & & u^* \end{array} \qquad (14)$$

$$\begin{array}{ccccc} \omega & \to & e & + & e^+ \\ dd^* & & d & & d^* \end{array} \qquad (15)$$

would suggest the relationship which might exist between leptons and quarks as particles built, according to Table 1, from the same Dirac states: $e \sim d$, $e^* \sim d^*$, $\nu_e \sim u$ and $\upsilon_e^* \sim u^*$. As may be seen in Table 1, there is at least one quark d having an electric charge like the electron and at least one u quark which doesn't carry any electric charge like the neutrino. This quark ~ lepton connection and the assumed structures for $s = u^*ud$ and $s^* = uu^*d$ quarks are revealed by the following decays:

$$\begin{array}{ccccccc} \Xi^- & \to & \Lambda & + & e & + & \upsilon_e^* \\ ds(s) & & ds(u) & & (d) & & (u^*) \end{array} \qquad (16)$$

$$\begin{array}{ccccccc} \Sigma^- & \to & n & + & e & + & \upsilon_e^* \\ dd(s) & & dd(u) & & (d) & & (u^*) \end{array} \qquad (17)$$

$$\begin{array}{ccccccc} \Lambda & \to & p & + & e & + & \upsilon_e^* \\ ud(s) & & ud(u) & & (d) & & (u^*) \end{array} \qquad (18)$$

$$\begin{array}{ccccccc} K^+ & \to & \pi^0 & + & e^+ & + & \upsilon_e \\ u(s) & & u(u^*) & & (d) & & (u) \end{array} \qquad (19)$$

This result is interesting. It suggests the possibility to describe the quarks (s,c) and (b,t) from the next two generations, obviously more massive and generating much more complicated decays than the d and u quarks, in terms of d and u quarks of the first family.

References:

(1) A.R.Lee and J.Liesegang, A semi-classical picture of the electron spin, *Physics Letters*, **37A** (1) 37-38 (1971).

(2) C. Lepadatu, "Quantum Theory as a Space - Time Theory", J. of Theoretics (Extensive Papers Section, 2002).

(3) C. Lepadatu, "Spin and Relativity", J. of Theoretics, Vol. 5-1(Feb. 2003).

(4) M. Y. Han and Y.Nambu, “Three-Triplet Model with Double *SU*(3) Symmetry”, Phys. Rev. **139**, B1006-B1010 (1965).

(5) F.E.Close, *An Introduction to Quarks and Partons* (Acad. Press, London, (1979).

(6) F.Halzen and A.D.Martin, *Quarks and Leptons* (J. Wiley & Sons, New York, 1979).

(7) H.M.Pilkhun, *Relativistic Particle Physics* (Springer Verlag, Berlin, 1979).

Received December 2002.

Journal Home Page