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A New Physical Model for the Atomic Mass Calculation

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Abstract – According to generally accepted physical model, the synthesis of the heavy elements may happen in the supernova explosions at a very high temperature. As a consequence of nuclear fusion, the supernova stars emit a very strong electromagnetic (EM) radiation dominantly in form of gamma rays and X-rays. The intensive EM radiation drastically decreases the masses of the exploding stars, directly causing mass defects of the nuclei. The general description of the black body EM radiation is based on the Planck's famous radiation theory, which supposes the existence of independent quantum oscillators inside the black body. In this paper, it is supposed that in exploding supernova stars, the EM radiating oscillators can be identified with the nascent heavy elements loosing their specific yields of own masses in the radiation process. The final binding energy of the nuclei is determined additionally by the strong neutrino radiation, which also follows the Maxwell-Boltzmann distribution in extremely high temperature. Extending to very high temperatures of the Planck's EM radiation law for discrete radiation energies, a very simple formula has been obtained for the description of the measured neutral atomic masses.
Keywords: atomic mass, Planck's radiation law, the origin of the elements, nuclei binding energy, nuclear synthesis model.

1. Introduction

Historically the theoretical determination of the neutral atom masses dates back to the year 1935, when C.F. von Weizsäcker [1] published his famous *liquid drop model* for the calculation of the nuclei masses. The model provides a general overview of masses and related stability of nuclei, and assumes the nucleus behaves in a gross collective manner, similar to a charged drop of liquid. The *semi-empirical mass formula* based on this phenomenological model successfully was applied mainly in the earlier period of the nuclear physics. From the simple drop model one can easily calculate approximately the mass of a neutral atoms by adding the Z number electron masses to the mass of $X(A,Z)$ nuclei. However, the liquid drop model does not give answers for many important questions related to the structure and forces inside the nuclei. The long-standing goal of nuclear physics has been to understand how the structure of nuclei arises from the interactions between the nucleons. It is known that nucleons are composed of quarks, but nuclear interactions have not yet been successfully derived from the fundamental interactions between the quarks. The standard way of modern calculation for light nuclei is based mainly on the non-relativistic quantum mechanics. In the world, many realistic phenomenological models of two- and three-nucleon interactions have been developed by fits to nucleon-nucleon (NN) scattering data and the properties mainly of ^2H, ^3H, and ^4He. The used non-relativistic Hamiltonian usually contains two-body and three-body potentials. Different types of approximation method are now available in the literature to solve the few-body problems; nevertheless, the fitting of the experimental data is remained necessary to date.

In this paper, we had no intention to investigate any further the nuclear structure and forces avoiding the difficult theoretical treatments and calculations. We only have focused to the birth of the nuclei in stars and given a very simple physical model for it. It is widely known that the main part of the elements of periodic table is synthesized in the stars. The synthesis of the heavy elements may happen only in supernova star explosions in very high temperature. In consequence of the nuclear fusion, the supernova stars and of course, the ordinary stars emit a very strong electromagnetic (EM) radiation dominantly in form of gamma and X-rays. In addition, the EM radiation is combined with strong neutrino radiation, which also follows the Maxwell-Boltzmann distribution in the case of extremely high temperatures. The intense energy radiation continuously decreases the masses of the stars directly, thus causing the mass defects of the nascent nuclei and the strong binding of nuclei. The individual nuclei represent quantitized black body oscillators; their frequencies are determined by their mass numbers. From this simple physical model, one can conclude that the binding energy curve of the nuclei is in immediate connection to Planck's radiation law in the high temperature region. In our paper, we have fitted Planck's radiation law to the binding energy curve of the nuclei, supposing that the radiation frequency of an arbitrary nucleon is proportional to the root-square of its mass number.

2. Extension of Planck's Radiation Law

According to the Planck's radiation law the energy density of the EM radiation in function of the radiation frequency is:

$$d\varepsilon = \frac{8\pi h}{c^3}\frac{f^3}{e^{hf/kT}-1}df\ , \tag{1}$$

where T is the absolute temperature, c is the speed of light, k is the Boltzmann constant, h is the Planck's constant, and f is the radiation frequency. Our model for the explanation of the origin of the elements requires discrete radiation frequencies of the stars depending on the mass numbers of the nuclei. In the classical electrodynamics, the radiation energy density of a simple dipole antenna is proportional to the frequency's fourth power:

$$\varepsilon_f = cst.\times f^4\ . \tag{2}$$

From the analogy the discrete energy emitted by the individual nuclei at T absolute temperature must be:

$$E_{rad}(A,Z) = cst.\times\frac{f^4}{e^{hf/kT}-1};\quad f = f(A,Z). \tag{3}$$

The binding energy of the nuclei *X(A,Z)* is equal to the negative value of the emitted energy. We can suppose that expression (3) is a natural generalization of Planck's radiation law for discrete radiation frequencies.

The most important task was to determine the mathematical relation between the radiation frequency and the arbitrary nucleon signed *X(A,Z)*. Our original goal was to calculate the neutral atomic mass values; therefore we have supposed that the radiation frequency mainly depends on its mass, i.e. on the mass number *A*. We have thought that the *Z* dependence may be very small in the frequency expression and it can take account later as a small correction.

Our working hypothesis was that the classical oscillator energy is proportional to the square of its frequency at constant amplitude which is shown as

$$E_{osc} = cst. \times f^2 , \tag{4}$$

and therefore the binding energy of a nucleon must be nearly proportional to the mass of the nucleon:

$$f^2(A) = cst. \times A \ . \tag{5}$$

3. The new mass-formula for the neutral atoms

Neglecting the dependence on Z of the neutral atoms, the mass calculation is based on the next simple expression

$$M(A) = AM_0 - E_{rad}(A)/c^2 , \tag{6}$$

where M_0 is a phenomenological constant and E_{rad} is the emitted energy from the star related to the nucleon with mass number A. The detailed form of (6) gives

$$M(A) = AM_0 - c_1 \frac{f^4}{e^{hf/kT} - 1}; \tag{7a}$$

where we have supposed by (5):

$$f = f(A) = c_2 \sqrt{A} \ . \tag{7b}$$

Replacing new variables for the unknown constants c_1 and c_2, we have

$$M(A) = M_0 \left(A - \frac{1}{2} \frac{\lambda F^4}{\eta^F - 1} \right); \ \ F = \sqrt{A} , \tag{8}$$

where λ, η and M_0 are the new fitting parameters for the experimentally determined atomic mass values. Here we introduced a new variable F proportioning to the radiation frequency.

4. Numerical results

In the numerical procedure we have observed, that the parameter η precisely equal to an important mass ratio:

$$\lambda = \frac{m}{M} = \frac{Electron\ mass}{Proton\ mass} , \tag{9}$$

and therefore the mass formula will be:

$$M(A) = M_0 \left(A - \frac{m}{2M} \times \frac{F^4}{\eta^F - 1} \right). \tag{10}$$

For the variable F defined in (8), we have found a better expression

$$F = \sqrt{A - 2} , \tag{11}$$

which means that the binding energy of the deuteron is very small, so its radiation frequency is approximately zero compared to heavier nuclei. Equations (10) and (11) yield excellent agreement with the average trend of the measured masses for all stable atoms *except those of very small A*. For this reason we have tried to decrease the values F^4 for the small A values. After some investigation, we have obtained a very successful solution:

$$F^4 = A(A-6) \ . \tag{11}$$

Finally, the new mass formula can be written:

$$M(A) = AM_0\left(1 - \frac{m}{2M} \times \frac{A-6}{\eta^{\sqrt{A-2}} - 1}\right) \ , \tag{12}$$

which involves only two fits: M_0 and η.

The introduced new mass formula represented by the (12) was fitted to near 2000 measured neutral atomic masses as obtained from the publication of G.Audi and A.H.Wapstra.[2]

The results of the fitting procedure are:

$$\begin{aligned} M_0 &= 1.003303846 \ a.u. \\ \eta &= 1.220126766 \end{aligned} \quad . \tag{11}$$

The accuracy of the new formula was determined by the relative standard deviation

$$\sigma = \sqrt{\frac{1}{N}\sum\left(\frac{M_{calc} - M_{mes}}{M_{mes}}\right)^2} = 3.216 \times 10^{-4} \ , \tag{12}$$

which yields excellent results. Figure 1 shows the relative errors of the fitted atomic masses.

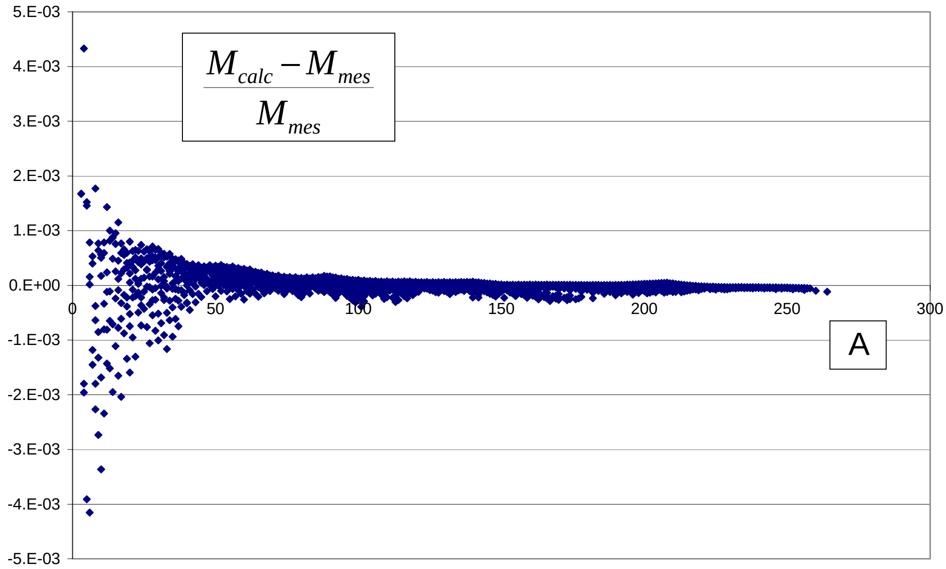

Figure 1. Relative deviations between calculated and measured atomic masses

5. Conclusions

Based on our new successful atomic mass formula we have concluded that in extreme high temperatures, the nuclear synthesis can be physically described exclusively following Planck's radiation law. In the Planck model it is supposed that the black body oscillators are independent to each other and have Maxwell-Boltzmann energy distribution. Already in the earlier nuclear physics, there were some experiences showing that all the nuclei inside the atom are weekly bound. This experimental fact was also proved theoretically here in the present work.

The accuracy of the introduced double-parameterized formula is comparative with the accuracy of the five-parameterized liquid drop model established by von Weizsäcker. Nevertheless, our model has a serious chance to improve its accuracy taking account additionally the Z-dependence of the atomic masses or more parameters (i.e., nucleon-spin, parity, etc.).

References

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