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Electronic Stopping Power at Low Energies

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Abstract: In our model we have used the Langevin equation in order to study the interaction of a charged particle ($M_p/m_e << 1$) with a gas of electrons. We have deduced an analytic expression for the electronic stopping power and the corresponding range. We have introduced a new concept that of the electronic friction and the deduction of its analytic expression. Our model works at low energies where the Bethe formula is not applicable.
Keywords: Transport equation, energy dissipation, stopping power.

INTRODUCTION

The transport equations are known to be a useful tool in the field of nuclear physics (i.e., fusion, fission, multifragmentation).1-4 We have therefore obtained an estimation of the friction present in the nuclear fluid, which is an important physical quantity. Similarly, we think that the atomic environment (constituted by the gas of electrons) possess a similar magnitude responsible for the energy dissipation of the incident particle. Because the slowing down of the charged particles when they pass through the matter is governed by their electromagnetic interaction with the electrons of the atomic environment, the slowing down of the particles is caused by inelastic interactions during which atoms are excited. The Brownian particle loses energy continuously in random walk until it is relaxed and totally stopped by the environment.

We are face in front of a relaxation phenomenon in which dissipation of energy occurs. We propose to apply a transport equation in order to study this process.

Macroscopic transport theories are usually based on the assumption that the physical system under consideration evolves on several well-separated time scales; the transport equation then describes the effective dynamics of only the slow degrees of freedom. Any ab-initio derivation of transport equations thus requires first an analysis, and then the optimal exploitation, of disparate time scales. This can be done very efficiently with the help of the so-called Nakajima-Zwanzig projection technique, which splits the space of physical observables into slow- and fast-evolving subspaces. Its main strength lies in the fact that by mapping the influence of irrelevant degrees of freedom onto (among other features) a non-local behavior in time it opens the way to the exploitation of separated time scales and so serves as a good starting point for powerful approximations, such as the Markovian and quasistationary limits; furthermore, it permits one to discern easily the dissipative and non-dissipative parts of the effective dynamics.

DYNAMICS

We consider the one-dimensional problem of a Brownian particle of mass *m* and charge *z* moving in an electron gas (no condition is being made concerning the gas). In one dimension, the only change of direction is backwards; it is an event negligible because the particle is heavy with respect to the electron mass. So it continues her route ahead.

We write the equation of motion of non-relativistic particle as follows:

$$\frac{dx}{dt} = \frac{p}{m} \tag{1-a}$$

$$\frac{dp}{dt} = -\frac{dV}{dx} - \beta p + L(t) \tag{1-b}$$

With the initial conditions $x(0)=0$ and $p(0)=p_0$.

p is the momentum of the particle, *x* its abscissa.

m : rest mass of the particle.

V : conservative interaction potential.

L(t) : Langevin force defined by its two first moments :

$$\langle L(t) \rangle = 0 \tag{2-a}$$

$$\langle L(t)L(t') \rangle = 2D\delta(t-t') \tag{2-b}$$

β : reduced friction

D : diffusion coefficient.

STOPPING POWER AND RANGE

In order to get the range and the stopping power of the particle, which are mean values, we take the mean values of the variables *x* and *p*. The fluctuating force vanishes under a statistical averaging and we obtain the Newton equations with a friction force. If we call *E* the total energy of the incident particle, *T* its kinetic energy, we can write from (1-a) and (1-b) the following identity:

$$-\frac{dE}{dx} = -\left(\frac{dT}{dx} + \frac{dV}{dx}\right) = -\left(\frac{dp}{dt} + \frac{dV}{dx}\right) = \beta p \tag{3}$$

According to the definition of the range we obtain:

$$\langle R \rangle = \int_{T_0}^{0} \left(-\frac{dE}{dx}\right)^{-1} dE = \frac{p_0}{m\beta} \tag{4}$$

In order to get the straggling, we have to solve the equations (1-a) and (1-b) numerically with the fluctuating term. We can get for large times $\left(t >> \beta^{-1}\right)$:

$$\left\langle \Delta R^2 \right\rangle = \left\langle R^2(t) \right\rangle - \left\langle R \right\rangle^2 = \frac{2T}{m\beta} t_{\max} - \frac{p_0{}^2}{m^2 \beta^2} \qquad (5)$$

T : electron gas temperature.

t_{max}: time over which $p(t)=0$.

DETERMINATION OF β

We can determine β if we apply the boundary conditions at the top of the stopping power curve, *i.e* at $\left(v = z^{2/3} \frac{e^2}{\eta} \right)$. We equal equation (3) with Bethe formula, we obtain the following expression for the friction coefficient:

$$\beta = \frac{4\pi n_e \eta^3}{m_e e^2 m} \left[\ln\left(\frac{2 m_e z^{4/3} e^4}{\eta^2 I} \right) - \ln\left(1 - \frac{z^{4/3} e^4}{\eta^2 c^2} \right) - \frac{z^{4/3} e^4}{\eta^2 c^2} \right] \qquad (6)$$

n_e: electron density
m_e : electron mass
e : electron charge
I : ionization potential
c: light celerity

DISCUSSIONS AND CONCLUSIONS

As in references where the authors have assumed a free electron gas description, the calculated stopping power was proportional to the velocity.[5,6] This proportionality was confirmed in a recent work for protons and antiprotons, the authors fit the experimental data with the following relation:

$$-\frac{dE}{dx} = a v^b \qquad (7)$$

a, b are free parameters.[7] The resulting velocity exponents b are 0.96±0.14, 0.89±0.30, 1.00±0.17 and 0.81±0.14 for carbon, aluminum, nickel and gold respectively. This is in good agreement with the predicted exponent of 1 from our model and from the free electron gas model.

In conclusion, in the paper the master equation(Langevin equation) of motion at low energies $\left(v < z^{2/3} \frac{e^2}{\eta} \right)$ to study the interaction of a charged particle with matter was discussed. We have extrapolated the formula until this velocity following the large experimental data in order to get the analytic expression for the friction coefficient. We have deducted analytic expressions for the electronic stopping power and the corresponding range.

REFERENCES

1. P. Grangé, H.A. Weidenmuller, Phys. Lett. **96B** (1980)26.

2. P. Frobrich, S.Y. Xu, Nucl. Phys. **A477** (1988)143.

3. S.P.Moller, A.Csete, T.Ichioka, H.Knudsen, U.I.Uggerhoj and H.H. Anderson, Phys. Rev. Lett. **88**,1 (2002).

4. M. M. R. Williams, Nuclear Science and Engineering, **136**, (2000), pp 34-58.

5. E.Fermi and E.Teller, Phys.Rev.**72**,399 (1947).

6. J. Lindhard, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. **28**, N°8 (1954).

7. M. M. R. Williams, Progress in Nuclear Energy, **36**, (2000), pp 239-322.

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