Journal of Theoretics
Volume 5-5, Oct-Nov 2003

Liquid Drop Parameters for a Cold Symmetric Nucleus

A. Zerarka azerarka@hotmail.com
K. Mahboub
V. G. Foester

Université Med Khider
Labo de Physique Théorique et Nucléaire,
BP 145, Biskra 07000, Algeria

Abstract: We have considered the nucleus as a degenerate Fermi gas, we have calculated the pressure of the gas on the moving surface of the nucleus. For symmetric spherical nucleus, we have expand the binding energy in decreased powers of the mass number A and reproduce the Bethe formula by taking the surface velocity proportional to 1/r. The volume-energy coefficient is calculated straightly and depends on the nuclear radius r_0 . The other coefficients a_s, a_{courb}, and a_0 depend on one parameter α which we determine by suggesting a value of one of the above coefficients. The curvature term a_{courb} is equal to 4.479. The parameter a_0 (A^0 term) is weak and equal to 0.54. These values are to be compared with those calculated in the Hartree-Fock theory, the FRDM and FRLDM.
Keywords: Binding energy, fission, volume-energy, liquid drop model.

I. INTRODUCTION

The Bethe semi-empiric formula [1] has shown a great success in describing dynamical phenomena (fission) and static properties of nuclei (fission barriers, ground-state masses). Different microscopic models (Hartree-Fock theory, semi-classical calculations) have been suggested to reproduce such this formula. This last one has been proved, but the disagreement between the different approaches lies in the various values of the parameters that enter the expression of the binding energy:

For example the curvature coefficient is 0 in the phenomenological models (FRLDM: finite range liquid drop model and FRDM : finite range droplet model) [2] and different of 0 but weak in the Hartree-Fock theory [3], range from 5.2 to 12.99 in the semi-classical approaches [4-6]. The same is for the A^0 term, the semi-classical calculations [7] suggest a value of 8.0 while the phenomenological models of FRDM suggest a value of 0 and the FRLDM gives a value of 2.615 .We attempt to contribute to a certain number of results by suggesting a simple model built on the Fermi gas and the dynamic evolution of the nuclear surface.

II. BINDING ENERGY OF THE NUCLEUS

The binding energy corresponds to the energy supplied to the nucleus in order to dissociate it from its constituents from the rest. This is equivalent to say that the nucleus volume has increased from its initial value V_0 to infinity. Let's imagine a process in which the work needed for this change corresponds to the binding energy of the nucleus. Let's call *P* the pressure exerted by the nucleons on the nuclear surface, and we assume that the nucleons possess the required work to increase the volume to infinity, it follows:

$$\Delta W = \int_{V_0}^{\infty} P\,dV \qquad (1)$$

The pressure exerted by the nucleons on a moving surface with a velocity $\dot{\varphi}$ in the z direction is:

$$P = 2\rho_* \int_{\dot{\varphi}}^{\infty} (v_z - \dot{\varphi})^2 g(v_z)\,dv_z \qquad (2)$$

ρ_* : mass density $=mN/V$

N : nucleon number, V : nucleus volume, m : nucleon mass

v_z : the velocity component according to z axis

$dW_z = g(v_z)dv_z = dv_z \iint f(v)dv_x dv_y$: proportion of nuclei having the third velocity component between $[v_z, v_z + dv_z]$.

f(v) : velocity function distribution of the particles.

All calculations made we found [8]:

$$P = \frac{1}{3}\rho_* \langle v\rangle^2 - \rho^* \langle v\rangle \dot{\varphi} + \rho^* \dot{\varphi}^2 - \frac{1}{3}\rho^* \langle v\rangle^{-1} + \Delta P \qquad (3)$$

$\langle v\rangle^n$: the n[th] mean velocity moment. For a degenerate Fermi gas (which is the case for the nucleus) it is given by :

$$\langle v\rangle^n = \frac{3}{n+3} v_f \qquad (4)$$

v_f : Fermi velocity. Because we have two kinds of particles then :

$$P = P_N + P_Z \qquad (5)$$

P_N , P_Z : pressure exerted by the neutrons, protons respectively.

II.1. Volume Term

We consider the case of symmetric nuclear matter *N=Z=A/2* and we take one kind of particles (protons). The integration of the first term of equation (3) gives:

$$W_P^1 = \int_{V_0}^{\infty} \frac{m_p A}{6V} \frac{3\eta^2}{5m_P^2} \left(\frac{3\pi^2 A}{2V}\right)^{2/3} dV = \frac{3\eta^2 c^2}{20 r_0^2 m_p c^2} \left(\frac{9\pi}{8}\right)^{2/3} A \qquad (6)$$

$$W^1 = W_P^1 + W_N^1 = \frac{3\eta^2 c^2}{20 r_0^2} \left(\frac{9\pi}{8}\right)^{2/3} \left(\frac{1}{m_p c^2} + \frac{1}{m_n c^2}\right) A = a_v A \qquad (7)$$

The volume term depends only on r_0 and is calculable analytically. In Table 1 we present some values of the parameter a_v for the different allowed values of the nuclear radius.

Table 1. Values of a_v for different values of r_0.

r_0 (fm)	a_v(MeV)
1.20	20.047
1.32	16.568
1.34	16.077
1.36	15.607
1.37	15.38
1.40	14.728

II.2. Surface term

To evaluate the other integrals we need to know the velocity $\dot{r}$ of the nuclear surface considered spherical at each instant of its evolution:

$$\dot{r}=\frac{dr}{dt}=\dot{r} \tag{8}$$

where $\dot{r}$ indicate the form of the radial velocity $\dot{r}$. For this we have two constraints:

1) The pressure must tend to zero when $V \to \infty$.
2) We must reproduce all decreasing powers of A and obtain the semi–empiric formula.

The expression of $\dot{r}$ which satisfies these conditions is :

$$\dot{r}=\frac{\alpha}{r} \tag{9}$$

We have $\dot{r}=\frac{\alpha}{r}$ tends to zero when $r \to \infty$ because we suppose that the surface stops at infinity.

The integration of the second term of equation (3) gives:

$$W^{(2)}=W_P^{(2)}+W_N^{(2)}=-\frac{3^{5/3}\pi^{1/3}\hbar\alpha}{4r_0^2}A^{2/3}=-a_sA^{2/3} \tag{10}$$

$W^{(2)}$ corresponds to the surface term which depends on two parameters r_0 and α .

II.3. Curvature term

The integration of the third term of equation (3) gives:

$$W^{(3)}=W_P^{(3)}+W_N^{(3)}=\frac{3\alpha^2}{4r_0^2c^2}\left(m_Pc^2+m_nc^2\right)A^{1/3}=a_{courb}A^{1/3} \tag{11}$$

This is the curvature term which depends also on the two above parameters.

II.4. A^0 term

The integration of the fourth term of equation (3) gives:

$$W^{(4)}=W_P^{(4)}+W_N^{(4)}=-\frac{3^{1/3}\alpha^3}{4\pi^{1/3}\hbar c^4r_0^2}\left(m_P^2c^4+m_n^2c^4\right)=-a_0 \tag{12}$$

It's the constant term, which is normally the last term of the expansion, it depends on r_0 and α . If we know the parameter α we can determine the three last ones for $\neq$ values of r_0. We can determine α by suggesting an experimental value of a_s and determine the two other coefficients directly .

In Table 2 we have evaluated the parameters a_{courb} and a_0 for different value of a_s from different models.

Table 2. Values of a_{courb} and a_0 for a given a_s (r_0=1.36 fm).

a_s (MeV)	a_{courb}(MeV)	a_0(MeV)
23.92	7.11	1.059
18.56	4.28	0.494
17.63	3.86	0.42

It remains the term ΔP which represents a small correction proportional to $A^{-2/3}$ for $\dot{\varphi} << v_f$.

III. CALCULATION OF THE BINDING ENERGY

In order to select the suitable coefficients we have calculated the binding energy for 7 symmetric nuclei. The energy denoted B_{exp} is calculated from the formula:

$$M_{exp}c^2 = Zm_pc^2 + Nm_nc^2 + Zm_ec^2 - B_{exp} \tag{13}$$

The calculated binding energy B_{calc} is given through the following formula :

$$B_{calc} = a_v A - a_s A^{2/3} - a_{coul}\frac{Z(Z-1)}{A^{1/3}} + a_{courb}A^{1/3} - a_0 \pm a_p A^{-1/2} \tag{14}$$

Where we have carefully added the coulomb and pairing energies. The best fit was obtained for the following set of coefficients (see Table 3 below).

Table 3. Cold liquid drop coefficients.

r_0(fm)	a_v(MeV)	a_s(MeV)	a_{courb}(MeV)	a_{coul}(MeV)	a_0(MeV)	a_p(MeV)
1.37	15.38	18.56	4.479	0.63	0.54	11

The value of a_{courb} is in agreement with those obtained by Hartree-Fock calculations. a_0 is weak to be compared with the value 0 of FRDM and 2.615 of FRLDM. For these coefficients we give the binding energy for the seven nuclei (see Table 4 below).

Table 4. Binding energies for different symmetric nuclei.

Nucleus	A	Z	B_{calc}(MeV)	B_{exp}(MeV)	$B_{exp} - B_{calc}$ (MeV)
He	4	2	26.028	28.296	2.268
Li	6	3	32.024	32.000	-0.024
Be	8	4	57.326	56.500	-0.826
O	16	8	127.718	127.620	-0.098
P	30	15	250.963	250.615	-0.348
K	38	19	323.252	320.650	-2.602
Ni	56	28	483.081	484.000	0.919

IV. CONCLUSION

We have deduced the Bethe formula for the binding energy where we have supposed the nuclear surface maneuver from its initial volume to infinity. We have found all the decreasing powers of A.
The curvature term is $\neq$ of 0 but is weak with respect to the semi-classical calculations. The a_0 term is weak in agreement with the phenomenological approaches FRDM and FRLDM.

We underline the fact that in our approach we have calculated the volume coefficient directly which depends only on r_0 while it is adjusted in the phenomenological approaches. Also the other coefficients depend on one parameter α and the knowledge of one among them allows the determination of the two others. We have only one free parameter in our model (α or a_s in our case).

The calculated binding energy was compared to the experimental one and the agreement is good for the majority of nuclei. We think that the introduction of shell effects must improve the results. The consideration of symmetric nuclei is not a restriction since we can add the asymmetric term by hand from any model or adjust its coefficient a_a to reproduce the experimental binding energies with all the other coefficients kept constant.

REFERENCES

[1] H.A.Bethe ,Phys Rev. **47** (1935) 747.
[2] P.Möller ,J.R.Nix ,WD .Meyers and W.J.Swiatecki ,to be published.
[3] W.Stocker ,M .Farine,Ann.of Phys.**159** (1985) 255.
[4] M.Brack ,C Guet and H-B Hakansson ,Phys.Reports **123**(1985) 275
[5] S. Aicher and M.Brack ,to be published.
[6] M.Dworzecka and S.Moszkowski ,Phys rev .**C12** (1975) 619.
[7] C.Guet ,E. Strumberger and M.Brack ,Phys.Lett .**B205** (1988) 427.
[8] J.Blocki ,Y.Boneh, J.R .Nix , J.Randrup , M. Robel , A.J. Sierk and W.J.Swiatecki Ann. of Phys .**113** (1978) 330.

Received 4/17/03

Journal Home Page