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The Pregeometric Scheme of Elementary Actions

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Abstract: This article is a direct address to J.A.Wheeler's assertion of the great theoretical interest of a pregeometrical approach to solve geometrodynamics singularities and simultaneously integrate quantum mechanics. We expose a formal theoretical pregeometrical scheme which represents the physical universe evolution by a random graph with binary labeled nodes inducing complex labeled arcs. A stochastic process governs the graph building, and rules can be given to extract geometry from this graph. For sake of brevity and clarity, only two questions are discussed : basic hypotheses are thoroughly described, and a bridge to geometry is given.
Keywords: geometrodynamics, pregeometry.

Introduction

In the chapter 44 of their great book on geometrodynamics [3] C.W.Misner, K.S.Thorne and J.A.Wheeler suggest that research is to be directed toward a theoretical scheme laid two levels under geometry, they call it pregeometry. At this time, Wheeler had some preliminary proposals, among them the "idea for an idea" of propositional logic as pregeometrical substrate to generate both quantum physics and geometrodynamics with an intervening reference to A.Sakharov's view of gravity as an elasticity in the particle's world.

Few pregeometrical approaches have been published, probably because too many useful concepts are to be put aside in the basic formalization and properly derived, which is a somewhat challenging task. One of them is the spin network invented by R.Penrose [7], refined by spin foams [1]. Some other tentatives to formalize discrete spacetime are relatives to those ideas [2]. All others tentatives to the generally called quantum gravity problem start from the space existence hypothesis with two directions: relativity and action of matter on space, or quantization. Used space-times are always continuous, and generally differentiable, except for L.Nottale [6] advocating fractal space-time.

The formal pregeometrical scheme we present has distinct advantages over previous approaches: the conceptual basis is both reduced and simple with no more mathematics than graph theoretical and stochastic process definitions. Beyond a real conceptual economy in basic concepts, developments in fact prove much richer than expected at first sight and expected enlightenments on quantum behavior and singular spacetime problematics do not appear out of reach. Finally, we have given elsewhere [4][5] some philosophical arguments that help in defining a firm and sound foundation for the choice of basic concepts.

A complete description of all presently seen consequences of this particular pregeometrical approach would both take too much space and be confusing; in the present text, we thus shall restrict our discussion to the thorough description of our model's hypotheses and method of geometrical bridging. The intent being to be as clear and precise as possible even if some aspects are at the moment passed under silence. In the same intent, all philosophical consideration will be put aside, and only the formal characters of the model are examined.

Basic Hypotheses and Comments

The following array of propositions is an axiom-like presentation of our hypotheses. No more than is contained in those propositions is to be added. We heavily insist on this point which is especially important, as every graphical representation adds artifactual aspects to the model, due to representation constraints on a plane (this will be the case in examples below). All those artifactually added properties must be mentally removed to only keep the properties that are contained in the propositions. Some length in the expression of those propositions is only a consequence of the desired precision. The described system is in fact extremely simple in its principle, though generated patterns are potentially extremely complicated.

Thus let us assert those propositions:

I) Substance postulate. The physical universe is composed of fundamental elements (thereafter called "acteons").

Acteons:

1) are atomic in the original sense of the term,
2) do preexist to any geometrical arrangement,
3) have a unique binary property (thus can have two states + or -),
4) can have their state changed.
5) have a continuous existence where absolutely nothing intervenes between two successive state changes.

II) Co-relation postulate. Acteons change their state by co-relation: an exclusive mutual action establishes, so that the state of each acteon flips in the complementary state. The name of elementary action is sometimes used. An acteon cannot co-relate with itself.

III) Probabilistic dynamic postulate. The dynamic development of co-relations obeys evolutive probabilistic laws. Those laws give at each universe alteration the probability for an acteon to co-relate with another distinct acteon.

Some comments are in order. First, acteons have no locus (they are pre-geometric), no mass (whether inertial or gravitational, no space thus no Newton laws), and no charge (no space thus no Coulomb law). In some sense, acteons are everywhere in a built space because if probability allows, an acteon can equally co-relate with acteons of very distant structures (acteons sets). By "built space" we mean space as we experience it, and built from the co-relation bundles between acteon sets, the emerging space.

They have no speed (pre-geometric entities), even if in a built space they can appear to manifest at intervals no shorter than a minimal change deduced from the stochastic evolution process in structures separated by a given possibly large distance, so that an apparent speed in built space can be calculated.

The third postulate should be taken with care as it is a "drawer postulate". In effect, it tells that dynamics is governed by probabilistic laws, but does not give the expressions of those laws. This point is to be precised in later work, but does not constitute a stumble block for the two objectives of the present work.

The Random Graph

In accordance with the above propositions, we represent the universe evolution as a stochastic process:

$$P = (U, U , P, A, \mathbf{N}, (X_t)).$$

The base space U is the set of all possible universes, U the □-algebra on U. P a probability measure on U giving the probability of a specific universe. The state space is A the set of all N×N adjacency matrix of diagonal degrees square, the process time (not to be confused with a physical time!) is the set of naturals, and all universes structures are contained in stochastic variables X_t.

The stochastic process produces a random graph which describes the realized evolution.

In fact, to effectively build the graph, it is more practical to give at each process step the matrix M which contains the probability $\square_i(j)$ for each acteon a_i to co-relate with another one a_j. From this probability matrix, a probability measure is induced on the space of adjacency matrix whose square is diagonal degrees matrix, and an adjacency matrix is drawn from this distribution, creating the graph layer with paths of unit length at this given step.

The real universe random graph is incredibly complex, being supported by the number of acteons which can be evaluated to the number of Planck volumes in the whole universe volume.

But to make the whole thing clear, let us build a very simple example with 4 acteons.

Example 1:

Our “universe” has N=4 acteons, a1, a2 a3, a4, and probabilities $\square_i(j)$ which govern the evolution are contained in the following matrix :

$$M = \begin{bmatrix} 0 & 0.8 & 0.1 & 0.1 \\ 0.8 & 0 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0 & 0.8 \\ 0.1 & 0.1 & 0.8 & 0 \end{bmatrix}$$

For example the acteon number 1 has probability 0.8 to co-relate with acteon 2, and probability 0.1 to co-relate with acteon 3. The matrix is symmetrical and kept for all process steps, but this is not necessary.

Values are chosen to evocate two particles in relation. Acteons 1 and 2 are the first particle and co-relate more in their set than out of it, acteons 3 and 4 the second with their own co-relations preference.

Now the probability for an oriented arc from acteon i to acteon j to occur is 1/N=1/4 times $\square_i(j)$ and the probability of the non oriented edge is $e_{ij}=(\square_i(j)+\square_j(i))/4$. For example the (1,2) edge has probability 0.4 and the (1,3) edge probability 0.05.

There are 9 null square adjacency matrix corresponding to the following graphs, their probability are calculated from the e_{ij} with the chained probabilities rule and given in the figure 1. (0.24=0.4(4x0.05+0.4) for the first graph for instance).

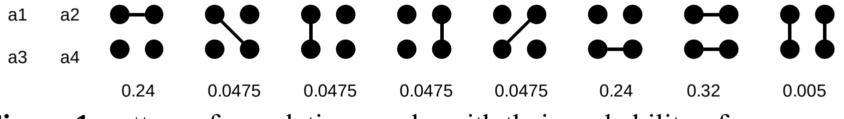

Figure 1. pattern of co-relation graphs with their probability of occurrence.

The evolution can be simulated with a random choice of those graphs according to their probability.

A short fragment of a trajectory of the process is given in figure 2 (remember that perspective and all aspects not contained in previously given hypotheses are irrelevant to the model).

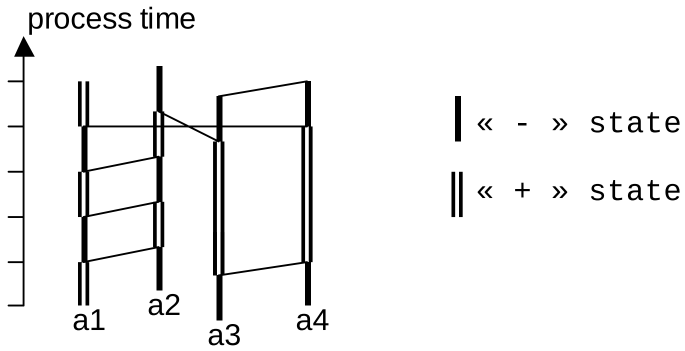

Figure 2. Five steps of the example evolution process on 4 acteons.

Null separations

Physical structures will be represented by intraverted sets of acteons, where we call intraverted an acteon set whose elements have a greater probability to co-relate inside the set than outside. The result being a greater density of internal actions and a lesser density of actions interesting exterior acteons. Structures thus appear as inhomogeneities in the evolution random graph.

Taken in its full generality and detail, the evolution process is much too complicated, that is highly fragmented or disordered (fractal), and a simplified approach is required. To achieve this, the simplest path is to consider great sets of acteons whose internal structure is neglected and only viewed as homogeneous with only mean properties. This kind of thermodynamical approach is privileged, as a first goal is to connect the acteons model with current theoretical pictures of physics which use continuous differential tools.

An important concept to get farther is the existential thickness of a structure. Given the acteon set of a structure S considered at evolution process step t, we define the existential thickness of S at t as the number of process steps necessary to flip all acteons of S at least once from t (a turnover number of steps).

We will thus interest ourselves in studying the co-relations bunches taken on an existential thickness of structures, those bunches being fully internal or in between structures. Geometry will arise from properties of those bunches of many co-relations.

Now observe that two state transitions are possible for acteons: + to - noted 0 and - to + noted 1. Co-relations then have four modalities, each noted with a unit module complex number :

$$(0\leftrightarrow 0)=1,\ (1\leftrightarrow 0)=j,\ (1\leftrightarrow 1)=-1,\ (0\leftrightarrow 1)=-j$$

Every co-relation bunch will thus be easily modelled by a complex number :

$$z=D_s+jD_a=(N_{00}-N_{11})+j(N_{10}-N_{01})$$

Where N_{00} is the number of bunch co-relation with modality 1 and so on. This means that we can have a mean simplified view of our full evolution graph which is another graph whose nodes are structures' acteon sets, and whose arcs materialize in between co-relation bunches. Nodes can be labelled with the complex number that represents the internal co-relation bunch of the structure, each arc has the complex number that represents the overall composition of the inter structure bunch.

If we make the reasonable assumption that arrival orders of co-relations modalities in a bunch are completely random along the evolution process time, we see that a co-relation bunch containing N co-relations is represented by a complex number whose modulus is the square root of N (a Brownian displacement).

Each bunch containing N co-relations (all modalities taken) is thus represented by the complex :

$$z=\sqrt{N}e^{j\theta}$$

Now if we want to qualify how a structure S_1 with internal bunch z_{i1} and proper external z_{x1} is related to a structure S_2 with internal bunch z_{i2} proper external z_{x2} and in between bunch z_{12}, we can take the inter bunch as reference for the internal bunches to form a complex vector of relative coefficients :

$$\eta^A=\frac{1}{z_{12}}\begin{bmatrix} z_{i1}+z_{x1} \\ z_{i2}+z_{x2} \end{bmatrix}=\frac{1}{\sqrt{N_{12}}}\begin{bmatrix} \sqrt{N_1}e^{j(\theta_1-\theta_{12})} \\ \sqrt{N_2}e^{j(\theta_2-\theta_{12})} \end{bmatrix}$$

As this method bears some similarity with relative rotation, we say this complex vector is a two component spinor (there is no hindrance if we can define the symmetry group effect in terms of graph transformations, which does not make problem).

So that we then apply the Penrose "null pole" formula :

$$u^\alpha=\begin{bmatrix} u^t \\ u^x \\ u^y \\ u^z \end{bmatrix}=(u^\alpha)^{AA'}=\begin{bmatrix} u^t+u^z & u^x+ju^y \\ u^x-ju^y & u^t-u^z \end{bmatrix}=\eta^A\eta^{*A'}=\begin{bmatrix} \eta^1\eta^{1*} & \eta^1\eta^{2*} \\ \eta^2\eta^{1*} & \eta^2\eta^{2*} \end{bmatrix}$$

Which, given the expression of z, becomes:

$$u^t=\frac{1}{2}\frac{N_1+N_2}{N_{12}} \quad u^z=\frac{1}{2}\frac{N_1-N_2}{N_{12}}$$

$$u^x=\frac{\sqrt{N_1N_2}}{N_{12}}\cos(\theta_1-\theta_2) \qquad u^y=\frac{\sqrt{N_1N_2}}{N_{12}}sin(\theta_1-\theta_2)$$

We thus can associate a null separation to each couple of structures, starting with counts on co-relation bundles and complex number representation.

Now this should be considered as a separation, not as a distance because distinction should be maintained between locus which is relationally defined (as above) and distance which is separation to overcome, and thus defined by a relational alteration process. The complete discussion of this point of view will be given in another text because is has developments related to quantum measurement defined as the process of structural integration of acteon structures.

This definition of separation allows the reproduction of a spacetime structure by an approach derived from the Regge calculus and its enhancements for gravitation simulators. A model spacetime can be given, test particles placed in it at uniformly chosen points, acteons sets are made to correspond to test particles, co-relation probabilities are calculated from 1/r laws between test particles, and the model spacetime can be forgotten because the graph built from probability laws reproduces the model. The use of a model space is a workaround because a probabilistic law for co-relations has not been given. This state of affairs is of course the next step to discuss: the form of the stochastic process probabilistic laws. But it would be out of present scope to place this discussion here, our sole objective being to relate the random graph to metric characteristics without any questions about the generation mechanism of the random graph.

Another toy example can be given to illustrate the method.

Example 2:

Let us take a small part of a 3 dimensional slice in a space-time as model giving x, y z distances to calculate corresponding acteons co-relation probabilities. Then let us chose five points in it so that they form the following figure where all polyhedral edges are supposed equal to d and the P1P2 distance h=kd is $(8/3)^{1/2}$ in absence of curvature. We will distribute 10 acteons only, so that a test particle of 2 acteons is associated to each point: acteons a1 and a2 in test particle at point P1, acteons a3 and a4 at P2 and so on:

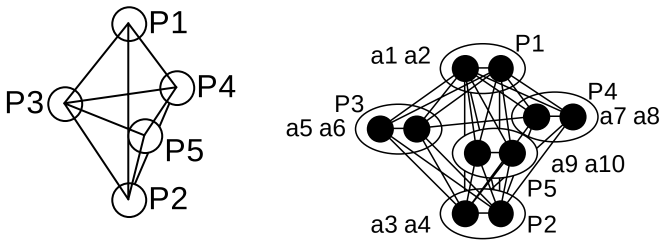

Figure 3. Model space disposition of five test particles containing 2 acteons and co-relation edges.

We know this figure to be interesting because curvature in a discrete 3D space is concentrated on 1D "hinges", here the hinge can be taken as the P1 P2 line, and the departure of k from $(8/3)^{1/2}$ will indicate curvature. The treatment is only done in 3D space to simplify, time intervals can be taken by difference and such figures can be piled along a time axis so that a tent like construction of space-time can be done as computed in simulators.

The problem has 15 unknowns which are the number or edges between two given points (five N_i and 10 N_{ij} for i from 1 to 5 and j > i). We have 10 equations from

$$N_3=N_4=N_5=n \quad N_1=N_2=\frac{2+k}{2-k}n \quad N_{12}=\frac{1}{kd}\frac{2+k}{(2-k)}n$$
$$N_{13}=N_{14}=N_{15}=N_{23}=N_{24}=N_{25}=\frac{2}{d(2+k)}n \quad N_{34}=N_{35}=N_{45}=\frac{n}{d}$$

u^t expressions and four from u^z which allow for a solution with one free parameter n :

From those expressions, we can deduce the edge probabilities, as there are many, let us give only some of them :

$$e_{12}=e_{34}=\frac{(2+k)d}{(4-k)d+24-3k} \quad e_{56}=e_{78}=e_{9\,10}=\frac{(2-k)d}{(4-k)d+24-3k} \quad e_{67}=e_{68}=e_{57}=e_{58}=\frac{1}{4}\frac{2-k}{(4-k)d+24-3k}$$

From those expressions a 10x10 matrix of $\square_{ij}$ can be built which govern a process reproducing separations. The model geometry can then be left aside (it was just there to take place of the probabilistic laws datum for the random dynamics postulate).

Next Directions Evocation

We intended to be short and precise on the two technical points above, but the reader will almost certainly wonder how other physical notions emerge, so that we hardly can avoid to suggest how some very important notions will be drawn from the graph. As shown just above, a spacetime can emerge from a random graph, embodying the idea of the relational definition of locus a processional definition of distances.

Then it appears that the general relativistic model of gravity as spacetime curvature arises from isolation of acteon structures (mass-energy is an effect of isolation that is the ratio of the number of internal co-relations on the number of co-relations going outside the structure).

Some study of the model shows that charge emerges similarly with the difference that isolation only concerns two modalities of co-relation on four possible.

The concept of field is absent of the pre-geometric graph model (which is one reason why it is somewhat difficult to cope with). In fact a field is conceived as the synthetic summary of the effects produced on a test particle which is supposed to be sequentially placed at each point of space where the field is present (this can only be done in imagination with a frozen field). In the graph model, where no particle is found, there is properly nothing, the vacuum does not exist independently of particles.

But once sources of the influence can be drawn from the graph (gravitational mass, electric charge), the field model can be derived. In the process, some logically implied physical aspects emerge as the necessary duality between our spacetime and a complementary one containing conjugated charges (antimatter), the notion of retroversion, or the distance-mass coupling, and add intriguing aspects to this fundamentally simple model which has some at first unexpected developments.

Conclusion

We have presented two technical points on how to start thinking of the physical universe as a random graph. We intended both to generate interest in the approach of thinking the universe in terms of a pre-geometric graph as described above, and to be precise enough by restricting the technical points examined. Of course, only developments will show the real interest of this conception, but present results sufficiently clear to be on the verge of publication give good confidence, and in fact the physical geometry derivation from a graph representation is an innovative point of view which deeply modifies the approach to several important physical concepts (distance as relational process or mass-energy as isolation, among other examples).

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