Journal of Theoretics
Volume 5-4, Aug-Sept 2003

Growth In α and the Bohr Radius

Rohitasch Tewari Rohitasch@rt.nl
SQ82 SFS, Hauz Khas, New Delhi 110016

Abstract: Recent work done on the implications and causes of the time-varying fine structure constant α investigate the possible relationships that α has with black hole entropy, the gravitational constant *G*, the speed of light *c*, quantum charge *e*, electron and proton masses and the Equivalence Principle. Here we look at α in terms of pure geometry and at face value interpret what its evolution implies in the simplest terms: Decay of the Bohr radius, though very slowly.
Keywords: Fine structure constant, atom.

The history of speculation and investigation into the causes and effects of the variation in the value of α is summed up by M.T. Murphy *et al.,* in [1]. What is needed to relate to here is listed below:

1. α^2 and α^4 in wave number
The dependence of the ratio of hyperfine and molecular rotational frequencies is shown to cause variations in $y \equiv \alpha^2 g_p$ by Murphy *et al.,* [2], where $g_p = \mu_P / \mu_N$ and J.K. Webb *et al* show that α is growing in magnitude [3] and the dependence of wave number on α is stated in general form as
$w_z = w_o + q_1 x + q_2 y$, where q_1 and q_2 quantify relativistic corrections for a particular atomic mass and electron configuration, $x = [(\alpha_z/\alpha_o)^2 - 1]$ and $y = [(\alpha_z/\alpha_o)^4 - 1]$ and $\Delta\alpha/\alpha = (\alpha_z - \alpha_o)/\alpha_o$.

2. Oklo Phenomena
Constraints put on $\Delta\alpha/\alpha$ from the natural reactor in Oklo is indirect as
$e^2/r_o \sim \alpha\, m_\pi c^2$ where r_o is the inter-nucleon separation and m_π is pion mass.

3. Reduced mass
The relation between α and reduced mass $m' = m_e m_p/(m_e+m_p)$ appears to suggest that the magnitude or reduced mass, or of mass itself decreases as α grows as shown by Gregori [5] and references therein.

4. Atomic clocks
The effect on atomic clocks is shown by Prestage *et al.,* [6] as a frequency drift between two clocks as a function of their respective atomic numbers.

5. Wipeout
Most recently, Alfonso-Faus logically shows in [7] that there is no way that a variation in α can be assumed to be a variation in *c* because *c* is not an explicit quantity in the $\alpha = e^2/h$ relationship. Moreover, the angular momentum quantum 'contains' quantities that are by their very nature invariant:

$$h = m\, \lambda_C\, c$$

Here *m* is the mass of any subatomic particle and λ_C is its Compton wavelength and since $m = \lambda_C^{-1}$,their

product is constant:

$$m_p \lambda_{C,p} = m_e \lambda_{C,e} = hc^{-1}$$

6. The common factor

The one thing common between all these approaches is the fact that the geometry of the atom itself is changing with α because any change in any of the quantities mentioned would affect the atomic structure itself. In fact any growth in α will ultimately result in a shortening of the Bohr radius a_o as seen here:

Photon wave number

$$\lambda^{-1} = R'_\infty = (\lambda_{c,e}/2\pi a_o)^2(\lambda_{c,p} + \lambda_{c,e})^{-1} = \alpha^2(\lambda_{c,p} + \lambda_{c,e})^{-1} \qquad (1).$$

λ_C is the Compton wavelength of the electron and proton respectively and R'_∞ is the mass reduced Rydberg constant.

Magnetic energy

$$E = \mu_B B = 0.5\, \mu_O e^2 \lambda_B v^2 (2\pi a_o)^{-2};$$

λ_B is the deBroglie wavelength of the electron at velocity v and B is the magnetic field of the proton from the electron's frame of reference in hydrogen.

$$E = 0.5\, \alpha^4 \lambda_{c,e} (\lambda_{c,p} + \lambda_{c,e})^{-1} m_e c^2 = 0.5\, \alpha^4 m_e c^2 \qquad (2).$$

The $\lambda_{c,e} (\lambda_{c,p} + \lambda_{c,e})^{-1}$ component replaces the usual $m_p/(m_e+m_p)$ for reduced mass.

(1) and (2) relate to the dependence of wave number on α as shown in section I and of reduced mass in section 3.

$$E (0.5\, m'c^2)^{-1} = \alpha^4$$
$$R'_\infty (\lambda_{c,p} + \lambda_{c,e}) = \alpha^2$$

Both these cases show that it is the Bohr radius that has decreased over a red shift z.

Next, we have the relation that shows the dependence of electrostatic force per electron mass across a_o on α:

$$F_e m_e^{-1} = a_{e/e} = v^2 a_o^{-1} = \alpha^2 c^2 a_o^{-1}$$
$$a_{e/e} = 2\pi c^2 \lambda_{c,e}^2 \lambda_B^{-3} = \alpha^3 2\pi \lambda_{c,e} v_{c,e}^2 \qquad (3).$$

Electrostatic force per proton mass across a_o simply replaces the electron's Compton wavelength with that of the proton:

$$a_{e/p} = 2\pi c^2 \lambda_{c,p}^2 (2\pi a_o)^{-3} = \alpha^3 2\pi \lambda_{c,p} v_{c,e}^2 \qquad (4).$$

Electrostatic acceleration is seen here to be dependent on the volume of the atom at the Bohr radius. An increase in the value of the fine structure constant will, as we can see, reduce the volume of the atom. The relationship mentioned in section 2 as $(e^2/r_o \sim \alpha\, m_\pi c^2)$ and the following relation of Compton wavelengths indicate that while the Bohr radius shrinks; the inter-nucleon distance does the same.

$$\lambda_{c,\pi} = (\mu_\pi)\, 0.5\, \alpha\, (\lambda_{c,p} + \lambda_{c,e}) \qquad (5).$$

The (μ_π) is the dimensionless mass factor analogous to the proton's g factor shown as g_p in section 1.

Very recently, Chiba and Khori worked out the dependence of luminosity on α [8] using SNIa observational information. They show that opacity of the universe changes in magnitude with α which results in smaller diffusion time for propagation of photons thereby making SNIa events with a smaller value for α more luminous. Diffusion time is given by

$$t_{diff} = \kappa\rho R^2/c \qquad (6).$$

Here κ is opacity (area/kg) and ρ is density (kg/volume). For a smaller t_{diff} eq.(6) suggests a smaller ρ so that a smaller α would mean a the emitter have a larger volume per kg of its mass.

We can see the occurrence of α in the atom with the following acceleration terms:
$a_1 = c^2/2\pi a_o = \alpha\, c^2/\lambda_{c,e}$
$a_2 = c^2 R_\infty = \alpha^2 c^2/\lambda_{c,e}$ (this is photonic acceleration of excitation)
$a_3 = v^2/a_o = \alpha^3 c^2\, 2\pi/\lambda_{c,e} = a_{c/e}$
$a_4 = v^3/ca_o = \alpha^4 c^2\, 2\pi/\lambda_{c,e}$ (this is magnetic acceleration).

Conclusion
Any way one looks at it, any increase in the magnitude of α will apparently decrease the distance between the electron orbits and the nucleus. This obviously decreases the time required for transmission of information within the atom (which would result in the drift in atomic clocks as is mentioned in section 4), and will be in harmony with the results of Chiba and Khori.

References

[1] M.T. Murphy *et al.*, *Mon. Not. R. Astron. Soc.*, 1 **17**(2000); astro-ph/0012419.

[2] M.T. Murphy *et al.*, *Mon. Not. R. Astron. Soc.*, 1 **6**(2001); astro-ph/0101519.

[3] J.K. Webb *et al.*, Sept 2001, http://xxx.lanl.gov/abs/astro-ph/0012539 .

[4] A.L. Shlyakhter, *Nature* **264**(1976) 340.

[5] Andrea Gregori, Nov 2002, http://xxx.lanl.gov/abs/hep-ph/0209296 .

[6] J.D. Prestage, R.L. Tjoelker, L. Maleki, *Phys. Rev. Lett.* **74**, 18, p3511.

[7] Antonio Alfonso-Faus, Dec 2002, http://xxx.lanl.gov/abs/gr-qc/0212020 and references therein.

[8] Takeshi Chiba, Kazunori Kohri, July 2003, http://xxx.lanl.gov/abs/astro-ph/0306486 .

Acknowledgements
I thank Taruna Sah, Vinay Kumar, Chetan Joshi, Saurabh Singh, Mayank and Mani.

Received 1/30/03
Updated on 7/18/03

Journal Home Page