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Pulse Scattering on Planes

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Abstract: *Integral equations recently developed to deal with Dirichlet and Neumann boundary value problems of the wave equation are used to investigate the scattering on perfectly conducting planes of transverse magnetic (TM) electromagnetic pulses with finite energy and duration. This approach is ex tended to mixed boundary value problems generated by scattering on impedance planes illustrated by a mirror coated with a thin layer of a dielectric.*
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1. Introduction

The scattering from obstacles of electromagnetic signals generated by modern technology in communications that consists of arbitrary time-dependent waves with finite duration and energy, is of a theoretical and practical interest because of a great variety of application domains including, remote sensing, radar technology, long range astronomy, surface physics, and so on. Causality informs us that not all of the points of the obstacle are reached simultaneously by the incident pulse, and therefore an important question is whether scattering introduces some distortion of the incident signal. One has in fact to deal with two situations accordingly as the obstacle reacts instantaneously or not to an electromagnetic excitation and only in the first case (for instance, for a perfectly conducting obstacle) may one expect that the pulse structure is left unchanged.

We investigate this problem by analyzing the scattering of electromagnetic signals when the obstacle S is a perfectly conducting or impedance plane. In the first case, the boundary conditions for the total field incident and scattered are of the Dirichlet or Neumann type while in the second case one has to deal with mixed boundary conditions. Assuming that the incident pulse is a TM field, solution of the 2D-wave equation, this investigation is made in the frame of the integral equation approach recently developed [1-3] to tackle boundary value problems of the scalar wave equation.

Then, this paper is organized as follows: Sec.2 is devoted to a presentation of the integral formulation used to deal with pulse scattering on planes. Sec.3 is concerned with signals impinging on perfectly reflecting planes and it is proved that the pulse structure is left unchanged, and the same problems are discussed in Sec.4 for impedance planes illustrated by a mirror coated with a thin layer of a dielectric. Conclusive comments are given in Sec.5.

2. Integral equations for pulse reflections: a different approach

A new integral equation approach [1-3] was recently proposed to deal with Dirichlet and

Neumann boundary value problems on planes for the scalar wave equation and applied to the scattering of TM and transverse electric (TE) electromagnetic fields on perfectly reflecting surfaces leading in fact to 2D- boundary value problems. To avoid later confusion, we make explicit some notations: **u**' = (x',z') is a source point in Green's functions and **u** = (x,z) an action point, · is the surface S (the plane z = 0) of action points and ·' the same surface S of source points; y(**u**,t) is the total field incident and scattered, G(**u**,t; **u**',t') a Green's function, and we impose on the ·-plane z = 0 on which scattering takes place the same boundary conditions for y and G, essentially of Dirichlet or Neumann type (soft or hard in acoustics)

$$[y(\mathbf{u},t)]_{z=0} = 0\ ,\ [G_D(\mathbf{u},t;\ \mathbf{u}',t')]_{z=0} = 0$$
$$[¶_z y(\mathbf{u},t)]_{z=0} = 0\ ,\ [¶_z G_N(\mathbf{u},t;\ \mathbf{u}',t')]_{z=0} = 0 \quad , \qquad (1)$$

in which $G_{D,N}$ are obtained by the method of images from the free space Green's function G, let **v** = (x,-z) denote the image point of **u** with respect to ·, G_D and G_N are defined by the relations

$$G_D(\mathbf{u},t';\ \mathbf{u}',t') = G(\mathbf{u},t;\ \mathbf{u}',t') - G(\mathbf{v},t;\ \mathbf{u}',t')$$
$$G_N(\mathbf{u},t;\ \mathbf{u}',t') = G(\mathbf{u},t;\ \mathbf{u}',t') + G(\mathbf{v},t;\ \mathbf{u}',t') \quad , \qquad (2)$$

while G is the inverse Laplace transformation of the Weyl representation of the Hankel function $H_0^{(1)}$

$$8i^{12}\ G(\mathbf{u},t;\ \mathbf{u}',t') = iº_{Br}\ ds\ º_{-∞}^{∞}\ dß\ s_z^{-1}\ N(ß,s)\ \exp[-s_z|z–z'|] \qquad (3)$$
$$N(ß,s) = \exp[s(ct-ct') = iß(x-x')] \quad , \qquad (3a)$$

in which $k_{sz} = (s^2 + ß^2)^{1/2}$ and the Bromwich contour Br is a line parallel to the imaginary axis of the complex s-plane with all the singularities of the integrand on its left.

We get from Green's theorem assumed to be valid for unbounded surfaces, the integral equation

$$y(\mathbf{u},t) = ºº_{-∞}^{∞}\ dx'\ dt'\ [G(\mathbf{u},t;\ \mathbf{u}',t')\ ¶_{z'}\ y(\mathbf{u}',t') - y(\mathbf{u}',t')\ ¶_{z'}\ G(\mathbf{u},t;\mathbf{u}',t')]_{z'=0}\ . \qquad (4)$$

Using in the integrand of (4) the boundary conditions $[y(\mathbf{u}',t')]_{z'=0} = 0$, $[¶_{z'}y(\mathbf{u}',t')]_{z'=0} = 0$ gives the two equations valid in the half space z ² 0

$$y_E(\mathbf{u},t) = ºº_{-∞}^{∞}\ dx'dt'\ \{[G_D(\mathbf{u},t;\ \mathbf{u}',t')\ ¶_z y_E(\mathbf{u}',t')]_{z'=0}$$
$$y_H(\mathbf{u},t) = -\ ºº_{-∞}^{∞}\ dx'dt'\ [¶_{z'} G_N(\mathbf{u},t;\ \mathbf{u}',t')\ y_H(\mathbf{u}',t')]_{z'=0} \quad , \qquad (5)$$

the subscripts E, H refer respectively to TE and TM electromagnetic fields and one obtains from (2) and (3) writing G+, G-, for GD, GN

$$8i^{12}\ G_{\pm}(\mathbf{u},t;\ \mathbf{u}',t') = iº_{Br}\ ds\ º_{-∞}^{∞}\ dß\ s_z^{-1}\ N(ß,s)\{\exp[-s_z|z–z'|] \pm \exp[-s_z|z+z'|]\} \quad , \qquad (6)$$

so that with z < 0 in |z-z'] and z > 0 in |z+z']

$$4i^{12}\ [G_D(\mathbf{u},t;\ \mathbf{u}',t')]_{z'=0} = º_{Br}\ ds\ º_{-∞}^{∞}\ dß\ s_z^{-1}\ N(ß,s)\ \sinh(s_z z)$$
$$4i^{12}\ [¶_z G_N(\mathbf{u},t;\mathbf{u}',t')]_{z'=0} = –\ º_{Br}\ dsº_{-∞}^{∞}\ dß\ N(ß,s)\ \cosh(s_z z) \quad . \qquad (6a)$$

Substituting (6a) into (5) and taking into account (3a) give for TE-waves, provided that exchanging x' and ß integrations is permissible

$$4i^{12}y_E(\mathbf{u},t) = {}^{o}{}_{Br}\, ds^{o}{}_{-\circ}dß\; s_z^{-1} \exp(sct + ißx) \sinh(s_z z)\, A_E(ß,s) \qquad (7)$$

$$A_E(ß,s) = {}^{oo}{}_{-\circ}\, dx'dt' \exp(-sct' - ißx')\, [¶_{z'}y_E(\mathbf{u}',t')]_{z'=0} \quad , \qquad (7a)$$

and for TM-waves

$$4i^{12}y_H(\mathbf{u},t) = {}^{o}{}_{Br}\, ds^{o}{}_{-\circ}dß \exp(sct + ißx) \cosh(s_z z)\, A_H(ß,s) \qquad (8)$$

$$A_H(ß,s) = {}^{oo}{}_{-\circ}\, dx'dt' \exp(-sct' - ißx')\, [y_H(\mathbf{u}',t')]_{z'=0} \quad . \qquad (8a)$$

To make clear the difference of the present integral approach with the conventional one, the comparison is made for harmonic fields. Then, Eqs.(7), (7a), become with $s_z = ik_z$

$$2^1 y_E(\mathbf{u}) = {}^{oo}{}_{-\circ}dß\; k_z^{-1} \exp(ißx) \sin(k_z z)\, A_E(ß) \qquad (9)$$

$${}_E(ß) = {}^{o}{}_{-\circ}\, dx' \exp(-ißx')\, [¶_{z'}y_E(\mathbf{u}')]_{z'=0} \quad . \qquad (9a)$$

In the conventional scattering theory [4], the total field $y_E(\mathbf{u})$ satisfying the Dirichlet boundary condition $[y_E(\mathbf{u})]_{z=0} = 0$ is given in terms of the incident field $y^i_E(\mathbf{u})$ by the integral relation

$$y_E(\mathbf{u}) = y^i_E(\mathbf{u}) - (1/2^1)^{o}{}_{-\circ}^{\circ}\, dx'\, H_0^{(1)}[k|\mathbf{u}-\mathbf{u}'|)\, [¶_{z'}y_E(\mathbf{u}')]_{z'=0} \quad , \qquad (10)$$

and to get the unknown normal derivative $[¶_{z'}y_E(\mathbf{u}')]_{z'=0}$ on ·' one has to solve the integral equation

$$[y^i_E(\mathbf{u})]_{z=0} = (1/2^1)^{o}{}_{-\circ}^{\circ}\, dx'\, \{H_0^{(1)}[k|\mathbf{u}-\mathbf{u}'|\}_{z=0}\, [¶_{z'}y_E(\mathbf{u}')]_{z'=0} \quad , \qquad (11)$$

obtained by letting **u** tend to · in (10) and by using the condition $[y_E(\mathbf{u})]_{z=0} = 0$. Both equations (10) and (11) in the conventional formulation take the place of the integral equation (9).

3. Pulse reflection on perfectly conducting planes

3.1 Normal incidence

We consider a pulse incident from the region z < 0 of space normally on the plane S

$$y_i(\mathbf{u},t) = f(t-z/c)\, [U(t-z/c) - U(t-t-z/c)] \quad , \qquad (12)$$

in which U is the unit step function, f an arbitrary function with partial derivatives while t is the duration of the incident pulse. Any point of S is reached by y_i at the same time t = 0 to which a reflected pulse $y_r(\mathbf{u},t)$ is generated so that one is only interested in the total field for t ³ 0 and from the Descartes-Snell law, we get

$$y_i(\mathbf{u},t) = f(t-z/c)\, U(t)\, [U(t-z/c) - U(t-t-z/c)] \qquad (13a)$$

$$y_r(\mathbf{u},t) = f(t-z/c) U(t)\, [U(t+z/c) - U(t-t+z/c)] \quad , \qquad (13b)$$

so that $y_E = y_i - y_r$, $y_H = y_i + y_r$, should be solution respectively of (7) and (8). Let us prove this statement for y_H and (8).

According to (13a,b) and discarding the useless repetition of the step function U(t'), one has

$$[y_H(\mathbf{u}',t')]_{z'=0} = 3f(t')\, [U(t') - U(t'-t] \quad , \qquad (14)$$

and substituting (14) into (8a) gives

$$A_H(\beta,s) = 2\int_{-\infty}^{\infty} dx' \exp(-i\beta x') \int_0^t dt' \, f(t') \exp(-sct') = 4\pi \, \delta(\beta)\int_0^t dt' \, f(t') \exp(-sct') \ , \quad (15)$$

in which δ is the Dirac distribution. Taking (15) into account, and exchanging the s and t' integrations, we get from (8) since $\beta = 0$ implies $s_z = s$

$$\psi_H(\mathbf{u},t) = (1/i\pi)\int_0^t dt' \, f(t') \int_{Br} ds \exp(sct - sct') \cosh(sz) = \int_0^t dt' \, f(t') \, [D_+(z,t') + D_-(z,t')] \ , \quad (16)$$

in which D is the inverse Laplace transformation supplied by the Bromwich integral

$$D_\pm(z,t = L^{-1}\{\exp[-sc(t'\pm z/c)]\} \quad , \quad (16a)$$

where we used the well known relation [5] $L^{-1}\{\exp(-as)\} = \delta(t-a)$ for $a > 0$. Substituting (16a) into (16) gives the expected result $\psi_H(\mathbf{u},t) = \psi_i(\mathbf{u},t) + \psi_r(\mathbf{u},t)$ since

$$\int_0^t dt' \, f(t') \, U(t'\pm z/c) \, \delta(t-t'\pm z/c) = f(t\pm z/c) \, U(t) \, [U(t\pm z:c) - U(t-\tau \pm z:c)] \quad , \quad (17)$$

the unit step functions in the square bracket coming from the fact that the integral on t' is non-null only if $t\pm z/c$ is in the interval $(0,\tau)$. So, a TM pulse with normal incidence reflects in agreement with the Descartes-Snell law.

3.2 Oblique incidence

For a scalar pulse impinging on the plane S with the incidence θ, the expression (12) becomes

$$\psi_i(\mathbf{u},t) = f(t-Z_i/c) \, [U(t-Z_i/c) - U(t-\tau-Z_i/c)] \ , \ Z_i = x \sin\theta + z \cos\theta \quad , \quad (18)$$

but now all the points of the plane S are not excited simultaneously and the reflected pulse is generated at the point x at the time $t = x \sin\theta/c$. So, we are interested in the total field along the vertical x for $t \geq x \sin\theta/c$ and still assuming that reflection brings no distortion, we write

$$\psi_i(\mathbf{u},t) = f(t-Z_i/c) \, U(t-x \sin\theta/c) \, [U(t-Z_i/c) - U(t-\tau-Z_i/c)] \quad (19a)$$
$$\psi_r(\mathbf{u},t) = f(t-Z_r/c) \, U(t-x \sin\theta/c) \, [U(t-Z_r/c) - U(t-\tau-Z_r/c)] \quad , \quad (19b)$$

with Z_r deduced from Z_i by changing $\cos\theta$ into $-\cos\theta$ and we have to prove that $\psi_i + \psi_i$ is solution of the integral equation (8). We get from (19, discarding the useless repetition of the step function U(t'-Z'/c)

$$[\psi_H(\mathbf{u}',t')]_{z'=0} = 2 \, f(t'-Z'/c) \, [U(t'-Z'/c) - U(t-\tau-Z'/c)] \ , \ Z' = x' \sin\theta \quad . \quad (20)$$

Then, substituting (20) into (8a) gives with Z'/c as lower bound and τ+Z'/c as upper bound of the t'-integral

$$A_H(\beta,s) = 2\int_{-\infty}^{\infty} dx' \exp(-i\beta x') \int dt' \exp(-sct') \, f(t'-Z'/c) \quad . \quad (21)$$

Introducing the variable r' = t'-Z'/c the integral (21) becomes

$$A_H(ß,s) = 2º_{-°}^{°} dx' \exp(-ißx') º_0^t dr' \exp(-scr') f(r')_i = 4^1 d(ß\text{-}is \sin q) º_0^t dr' f(r') \exp(-scr') . \quad (21a)$$

Substituting (21a) into (8) gives

$$i^1 y_H(\mathbf{u},t) = c\, º_0^t dr' f(r') \exp(-scr')\, º_{-°}^{°} dß\, d(ß\text{-}is \sin q) \exp(ißx) \cosh(s_z z) \quad . \qquad (22)$$

But from the definitions (18) of Z_i, Z_r, and since $s_z = (s^2+ß^2)^{1/2}$, we get

$$2º_{-°}^{°} dß\, d(ß\text{-}is \sin q) \exp(ißx) \cosh(s_z z) = \exp(-sZ_i) + \exp(-sZ_r) \quad . \qquad (23)$$

Substituting (23) into (22) gives

$$y_H(\mathbf{u},t) = º_0^t dr' f(r') [D_i(\mathbf{u},r') + D_r(\mathbf{u},r')] \qquad (24)$$

in which D_i and D_r are the inverse Laplace transform supplied by the Bromwich integrals

$$D_i(\mathbf{u},r') = L^{-1}\{\exp(-scr'-sZ)\}= d(t\text{-}x \sin q/c) - (r'+ z \cos q/c)]U(r'+ z \cos q/c) \qquad (25a)$$

in which we used the same propeerty of the Laplace transform as in Sec.3.1. Similarly

$$D_r(\mathbf{u},r') = d[(t\text{-}x \sin q/c) - (r'- z \cos q/c)]U(r'- z \cos q/c) \quad . \qquad (25b)$$

Substituting (25a,b) into (24) gives $y_H(\mathbf{u},t) = y_i(\mathbf{u},t) + y_r(\mathbf{u},t)$ since

$$º_0^t dr' f(r') [D_i(\mathbf{u},r')] = º_0^t dr' f(r') d[(t\text{-}x \sin q/c) - (r'+ z \cos q/c)]U(r'+ z \cos q/c)$$
$$= f(t-Z_i/c)\, U(t\text{-}x \sin q/c) [U(t-Z_i/c) - U(t\text{-}t\text{-}Z_i/c)] \quad , \qquad (26)$$

while for D_r one has just to change Z_i into Z_r on the right hand side of (26). So, for a TM pulse impinging at any angle on a perfectly conducting plane, reflection introduces no distortion and the reflected pulse is obtained according to the Descartes-Snell law although all the points are not excited simultaneously which is reminiscent of analytic functions: they are known along the entire axis (-°, °) as soon they are defined on any finite interval.

4. Pulse scattering on an impedance plane

We now investigate what happens on an impedance plane for a TM electromagnetic pulse satisfying instead of the Neumann boundary condition (1) the mixed boundary condition

$$[¶_z y(\mathbf{u},t) +Z\, y(\mathbf{u},t)]_{z=0} = 0 \ , \ [¶_z G_R(\mathbf{u},t;\, \mathbf{u}',t') +Z\, G_R(\mathbf{u},t;\, \mathbf{u}',t')]]_{z=0} = 0 \quad , \qquad (27)$$

and we suppose that the operator Z generalizing to time-dependent fields that on would obtain for a mirror coated with a thin layer of a dielectric [6, 7] is

$$Z = d(q)c^{-2}¶_t^2 \quad , \qquad (27a)$$

where d is a thickness depending on the angle of incidence q. Substituting (27a) into (27) and taking the Laplace transform, we get with δ written for $\delta(\theta)$

$$[\partial_z\psi^*(\mathbf{u},s) + s^2\delta\, \psi^*(\mathbf{u},s)\,]_{z=0} = 0\ ,\ [\partial_z\, G_R^*(\mathbf{u},s;\, \mathbf{u}',t') +s^2\delta\, G_R^*(\mathbf{u},s;\, \mathbf{u}',t')]\,]_{z=0} = 0 \qquad (28)$$

$$\psi^*(\mathbf{u},s) = L\{\psi(\mathbf{u},t)\}\ ,\ G_R^*(\mathbf{u},s;\, \mathbf{u}',t') = L\{G_R(\mathbf{u},t;\, \mathbf{u}',t')\} \quad . \qquad (28a)$$

Now according to (3a) and (6)

$$4\pi\, G_N^*(\mathbf{u},s;\, \mathbf{u}',t') = \int_{-\infty}^{\infty} d\beta\, s_z^{-1}\, M(\beta,s)\{\exp[-s_z|z-z'|] + \exp[-s_z|z+z'|]\}$$

$$4\pi\, G_D^*(\mathbf{u},s;\, \mathbf{u}',t') = \int_{-\infty}^{\infty} d\beta\, s_z^{-1}\, M(\beta,s)\{\exp[-s_z|z-z'|] - \exp[-s_z|z+z'|]\} \qquad (29)$$

$$M(\beta,s) = \exp[-sct' + i\beta(x-x')] \quad . \qquad (29a)$$

We prove in Appendix A that the Green's function with primitive operator $\partial_x^{-1} = \int dx$

$$G_R^*(\mathbf{u},s;\, \mathbf{u}',t') = G_N^*(\mathbf{u},s;\, \mathbf{u}',t') -s^2\delta\, \partial_{|z|}G_D^*(\mathbf{u},s;\, \mathbf{u}',t') \quad , \qquad (30)$$

satisfies the boundary condition (28) and that

$$2\pi\, [\partial_{z'}G_R^*(\mathbf{u},s;\, \mathbf{u}',t')]_{z'=0} = \int_{-\infty}^{\infty} d\beta\, M(\beta,s)\, [\cosh(s_z z) - (s^2\delta/s_z)\sinh(s_z z)] \quad . \qquad (31)$$

But we get from the second equation (5)

$$\psi_H^*(\mathbf{u},s) = - \iint_{-\infty}^{\infty} dx'dt'\, [\partial_{z'}G_N^*(\mathbf{u},s;\, \mathbf{u}',t')\, \psi_H(\mathbf{u}',t')]_{z'=0} \quad . \qquad (32)$$

Substituting (32) into (31) and using the definition (29a) of $M(\beta,s)$ give

$$2\pi\, \psi_H^*(\mathbf{u},s) = \int_{-\infty}^{\infty} d\beta\, \exp(i\beta x)[\cosh(s_z z) - (s^2\delta/s_z)\sinh(s_z z)]\, A_H(\beta,s) \quad , \qquad (33)$$

where $A_H(\beta,s)$ is the function (8a) and finally

$$4i\pi^2\, \psi_H(\mathbf{u},t) = \int_{Br} ds \int_{-\infty}^{\infty} d\beta\, \exp(sct+i\beta x)[\cosh(s_z z) - (s^2\delta/s_z)\sinh(s_z z)]\, A_H(\beta,s) \quad . \qquad (33a)$$

Let $\psi_{H,0}$,$\psi_{E,0}$, denote the solutions of the Neumann and Dirichlet boundary conditions discussed in Sec.3 and $A_{H,0}$,$A_{E,0}$, the corresponding expressions (8a), (7a). Then, if

$$A_{H,0}(\beta,s) = \lambda s^{-1}\, A_{HE0}(\beta,s) \quad , \qquad (34)$$

where λ is a constant parameter, the integral equation (33a) has the solution

$$\psi_H(\mathbf{u},t) = \psi_{H,0}(\mathbf{u},t) - \lambda\delta c^{-1}\partial_t\psi_{E,0}(\mathbf{u},t) \quad . \qquad (35)$$

Since $[\psi_H(\mathbf{u}',t')]_{z'=0} = [\psi_{H,0}(\mathbf{u}',t')]_{z'=0}$ according to (35), we get from (33) and from the inverse Laplace transform of (8)

$$2\pi\, \psi_H^*(\mathbf{u},s) = 2\pi\, \psi_H^*{}_0(\mathbf{u},s) - \lambda\delta s \int_{-\infty}^{\infty} d\beta\, \exp(i\beta x)s_z^{-1}\sinh(s_z z)]\, A_{H,0}(\beta,s) \quad . \qquad (36)$$

Substituting (34) into (36) and using the Laplace transform of (7) give

$$y_H{}^*(\mathbf{u},s) = y_{H,0}{}^*(\mathbf{u},s) - lsdy_{E,0}{}^*(\mathbf{u},s) \quad . \qquad (37)$$

The inverse Laplace transform of (37) gives (35). Then, with $y_H = y_i+y_s$, $y_{H,0} = y_i+y_r$, $y_{E,0} = y_i-y_s$, in which y_s is the field diffracted by the impedance plane and y_r the reflected field, we get from (35)

$$y_s(\mathbf{u},t) = y_r(\mathbf{u},t) - ldc^{-1}¶_t\,[y_i(\mathbf{u},t) - y_r(\mathbf{u},t)] \quad . \qquad (38)$$

The condition (34) is fulfilled by the pulse (18) and it is proved in Appendix B that $l = 1/\cos q$. The second term in (38) represents the pulse distortion due to scattering.

5. Discussion

The present investigation is mainly devoted to the case of an incident TM electromagnetic pulse and one would proceed similarly for a TE, but calculations would be a bit more elaborate. The impedance (27a) is a translation to time dependent fields of an impedance obtained by Idemen [6,7] for a coated planes and harmonic waves. Then $c^{-2}¶_t^{\,2}$ is changed into w^2 and

$$d(q) = de^{-1}(e-\sin^2 q) \quad , \qquad (39)$$

in which e, d, are the dielectric constant and the thickness of the coated sheet while q is the angle of incidence: generalizations of (39) may be found in [6,7]. The main virtue of (27a) is to supply a simple solution (35) to the integral equation (33), a result valid as long as Z does not depend upon the position (upon x here) on the obstacle which is not the case for pulse scattering on a rough plane when roughness may be described by an impedance: an important theoretical and practical problem, still to be investigated.

As said in the introduction, modern technology in communication is an incentive to analyze the transmission of electromagnetic pulses in various media and its scattering by obstacles. Thus, for instance, the difficult diffraction problem discussed a long time ago by Sommerfeld [8] and Brillouin [9] was correctly solved only recently [10]. On the other hand, Harmuth et Al.[11] have devoted a book to propagation and reflection of many different electromagnetic signals with a through discussion on the practical consequences of their results. They have a particular look at the Descartes-Snell law proved here to be valid for Dirichlet and Neumann boundary value problems while the expression (38) gives the distortion generated by a mixed boundary value problem.

So, the present integral equation approach is a suitable tool to investigate pulse scattering from obstacles: devoted here to planes, this formulation is easy to transpose to some simple surfaces as cylinders or paraboloids and more generally to any surface for which one may get the Green's functions of the wave equation, but generally in these cases one has to be satisfied with approximate solutions. Finally, the extension of these integral equations to 3D-boundary value problems for arbitrary electromagnetic pulses is trivial [1-3].

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Appendix A

We prove that the following Green's function in which $\partial_{|x|}^{-1} f(x) = \int f(x)\, dx$

$$G_R^*(\mathbf{u},s;\ \mathbf{u}',t') = G_N(\mathbf{u},s;\ \mathbf{u}',t')) - s^2 d\, \partial_{|z|}^{-1}\, G_D(\mathbf{u},s;\ \mathbf{u}',t') \quad . \tag{A.1}$$

satisfies the boundary condition

$$[\partial_z G_R^*(\mathbf{u},s;\ \mathbf{u}',t') + s^{é} d\, G_R^*(\mathbf{u},s\ \mathbf{u}',t')]\,]_{z=0} = 0 \quad . \tag{A.2}$$

From the definition (29) of G_N*and according to the relation $[\partial_z|z\pm z'|]_{z=0} = \pm z'/|z'|]$, we get

$$[\partial_z G_N^*(\mathbf{u},s;\ \mathbf{u}',t'))]_{z=0} = 0\ ,\ 2^1\, [G_N(\mathbf{u},s;\ \mathbf{u}'),t']\,]_{z=0} = \int_{-\infty}^{\infty} d\beta\, M(\beta,s)\cosh(s_z z)\ , \tag{A.3}$$

and still using (29)

$$4^1\, \partial_{|z|}^{-1} G_D^*(\mathbf{u},s;\ \mathbf{u}',t') = -\int_{-\infty}^{\infty} d\beta\, s_z^{-2} M(\beta,s)\{\exp(-s_z|z-z'|) - \exp(-s_z|z+z'|\}\quad , \tag{A.4}$$

so that

$$[\partial_{|z|}^{-1} G_D^*(\mathbf{u},s;\ \mathbf{u}',t')]_{z=0} = 0\ ,\ [\partial_z \partial_{|z|}^{-1} G_D^*(\mathbf{u},s;\ \mathbf{u}',t')]_{z=0} = [G_N^*(\mathbf{u},s;\ \mathbf{u}',t')]_{z=0}\ . \tag{A.5}$$

Taking into account (A.3) and (A.5) we get from (A.1)

$$[\partial_z G_R^*(\mathbf{u},s;\ \mathbf{u}',t')\,]_{z=0} = -s^2 d\, [G_N^*(\mathbf{u},s;\ \mathbf{u}',t')]_{z=0}$$

$$[G_R^*(\mathbf{u},s:\ \mathbf{u}',t')]\,]_{z=0} = [G_N{}^*(\mathbf{u},s;\ \mathbf{u}',t')]\,]_{z=0} \quad , \tag{A.6}$$

implying that G_R* satisfies (A.2).

Let us now consider $[\partial_{z'} G_R^*(\mathbf{u},s:\ \mathbf{u}',t')]\,]_{z'=0}$, we first remind that z < 0 for the action point in

$|z-z'|$ and $z > 0$ for its image in $|z+z'|$ so:

$$[z-z']_{z'=0} = -z \text{ for } z < 0\ ,\ |z+z'|_{z'=0} = z \text{ for } z > 0$$
$$[\partial_{z'}|z-z'|]_{z'=0} = [\partial|z+z'|]_{z'=0} = 1 \text{ for any } z \quad . \qquad \text{(A.7)}$$

So, using (A.7) we get from (A.4)

$$2^1\ [\partial_{z'}\partial_{|z|}{}^{-1} G_D{}^*(\mathbf{u},s;\ \mathbf{u}',t')\ ]_{z'=0} = \int_{-\infty}^{\infty} d\beta\ s_z{}^{-2} M(\beta,s) \sinh(s_z z) \quad . \qquad \text{(A.8)}$$

Substituting (A.8) and the Laplace transform of the expression (6a) into (A.1) gives

$$2^1\ [\partial_{z'} G_R{}^*(\mathbf{u},s;\ \mathbf{u}',t')\ ]_{z'=0} = \int_{-\infty}^{\infty} d\beta\ s_z{}^{-2} M(\beta,s)\ [\cosh(s_z z)-(s^2 d/s_z)\sinh(s_z z)] \quad . \qquad \text{(A.9)}$$

Appendix B

Since $\psi_e(\mathbf{u},t) = \psi_i(\mathbf{u},t) - \psi_r(\mathbf{u},t)$ we get from (19a,b) with $Z' = x' \sin\theta$

$$[\partial_{z'}\psi_i(\mathbf{u}',t')]_{z'=0} = 2 \cos\theta\ c^{-1}\partial_{t'}\ f(t'-Z'/c)\ [U(t'-Z'/c) - U(t'-\tau-Z'/c)]$$
$$- 2 \cos\theta\ c^{-1}\ f(t'-Z'/c)\ [\delta(t'-Z'/c) - \delta(t'-\tau-Z'/c)] \quad , \qquad \text{(B.1)}$$

and from the definition (7a) of A_E, one has to perform the integration $\int_{-\infty}^{\infty} dt'\ \exp(-sct')[\partial_{z'}\psi_E(\mathbf{u}',t')]_{z'=0}$ but with $a = Z'/c$, $b = \tau+Z'/c$

$$-2 \cos\theta\ c^{-1} \int_{-\infty}^{\infty} dt'\ \exp(-sct')\partial_{t'}\ f(t'-Z'/c)\ [U(t'-Z'/c) - U(t'-\tau-Z'/c)] =$$
$$- 2 \cos\theta\ c^{-1}\ \{f(t'-Z'/c) \exp(-sct')\}_a{}^b + 2s \cos\theta \int_a^b dt'\ \exp(-sct')\partial_{t'}\ f(t'-Z'/c) \qquad \text{(B.2)}$$

and

$$- 2 \cos\theta\ c^{-1} \int_{-\infty}^{\infty} dt'\ \exp(-sct')f(t'-Z'/c)\ [\delta(t'-Z'/c) - \delta(t'-\tau-Z'/c)] =$$
$$- 2 \cos\theta\ c^{-1}\ \{f(0) \exp(-sc') - f(\tau) \exp[-s(Z'+c\tau)]\} \qquad \text{(B.3)}$$

the sum of the two curly brackets in (B.2) and (B.3) is zero, so one is left with

$$\int_{-\infty}^{\infty} dt'\ \exp(-sct')[\partial_{z'}\psi_E(\mathbf{u}',t')]_{z'=0} = 2 \cos\theta\ c^{-1} \int_a^b dt'\ \exp(-sct')f(t'-Z'/c) \quad . \qquad \text{(B.4)}$$

Substituting (B.4) into (7a) and using (21) gives

$$A_{E0}(\beta,s) = s \cos\theta\ A_{H0}(\beta,s) \quad . \qquad \text{(B.5)}$$

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