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How can Bohr's Atomic Model Predict Hydrogen's Spectrum Line Spacing?

(The prediction of the hydrogen spectrum line spacings relative to the area that is generated from the magnetic and electric vectors composing the related frequencies specific to these lines.)

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Abstract: This article will present a prediction of hydrogen spectrum line spacings, their relation to the difference in the value of the surface that is generated from the product of the respective lengths of both the magnetic and electric vectors, which are specific to the emitted radiations as attached to their representation on the spectrum, lines being taken two by two .
Keywords: hydrogen, Bohr atomic model, hydrogen spectrum.

It is necessary to display the initial formula before applying its differentiation to the atom of hydrogen:

$$dF^{1.5} \cdot dr^2 \cdot m_e \; / \; h = Z = \text{constant} \quad .$$

This formula reads as: the product of the ionization frequency at power 1.5, that is attached to each of the two respective levels of energy, multiplying the distance (at power 2) that is attached to each of the two levels which have been considered and calculated from the Bohr's atomic model, thus multiplying the mass of an electron and dividing by the Plank's constant, yields a constant value, here called Z .

This equation is useful to characterize a jump between two respective levels of energy as long as the jumps are between equal differences in n. For instance, if an electron jumps from level 6 to level 2, the difference in n is 4, same as in a jump from level 5 to level 1. (The presence of the electronic mass to h quotient is due to one of the previous formulae which resulted in that one .)

For instance, these are the calculations for the jumps over a single level: jump 2-1, 4-3, 5-4, 6-5, 7-6, and 8-7 as shown below:

Between	$dF^{1.5}$ $.dr^2$	Z
2-1	1.88E23	7.26 E5
4-3	1.868E23	7.21E5
6-5	1.899E23	7.25E5
7-6	1.877E23	7.27E5
8-7	1.25E23	4.82E5

Before commenting on these results, it is important to clarify how the product [dF·dr] has to be calculated:

$$DF^{1.5} \cdot dr^2 = a_0^2 \left[\left(dF^{1.5}_{n+1} \cdot n^4 - dF^{1.5}_{n} \cdot n^4 \right) \right]$$

a = the Bohr's atom radius for the first level of energy, ie: 0.53E-10; dr is equal, for a considered level, to the radius of that level such as $a_0 \cdot n^2 = dr$, and in the formula, dr is elevated to the square and the value of the Bohr's radius is in factor.

This formula shows us that for a given specific jump that is characterized by the number of levels which are 'jumped over', no matter the value of the ionization frequencies or the radii of the levels of energy, the product [dF·dr], as explained, gives a constant value that depends strictly upon the number of levels which were jumped over. Here, the Z value was calculated for the Bohr atom and stops being a constant for the 8th energy level. (Note that the subscripts n+1 and n in [dF·dr] implies with a jump over one single level, but this could be changed if another jump were to be studied.

This formula is differentiated from a macro-scale formula, enabling to know exactly the underlying concepts that should help understanding the Z result: in this case, the distance between the given levels of energy are assimilated to the width of a potential well, the atom to an isolated system, and the matter to an homogeneous matter, offering constant relative permeability and magnetic properties.

We can now try to show and understand how the spacings in the spectrum of Hydrogen can be calculated. Firstly, the above formula can be differentiated again, but this time, I will use the values of the magnetic and electric vectors of emitted radiation that is recorded after the jump of electronic charges from a lower to a higher energy level.

But to calculate the magnetic vector, I still need the above equation, whereby the product [dF·dr] is equivalent to the product of the frequency of the emitted radiation by the length of the magnetic vector, which is left unknown such that this is the value to calculate in the first place. The full formula is :

$$F_{\lambda}^{1.5} \cdot r_{\beta}^{2} = dF_{E}^{1.5} \cdot r_{s}^{2}$$

This formula reads as if $F_{\lambda}^{1.5}$ is the value of the emitted radiation after the fall of an electron to a higher energy level, and if r_{β}^{2} is the value of the length of the magnetic component vector of that radiation, then their product is equal to [dF·dr] as seen before, wherein the r index s specifies that, over the period of the emitted radiation, the electron which could not reach the higher level of energy, falls back by a distance (i.e., s) from its culminating point and wherein the dF index E just reminds us that it is initially an incoming frequency which provided energy before the electron started to move. This is not changing anything to the already explained calculation of [dF·dr].

As a result, this formula makes it possible to retrieve the value of the length of the magnetic vector after changing, knowing the value of the emitted radiation frequency. I am now going to show that the surface that is generated from the product of the related magnetic vector by the electric vector of the emitted radiation, and their comparison two by two (that is the difference between the values two by two) is directly proportional to the spacings between the spectrum's lines, as you can see them as a result from diffraction. Simply, according to the diffraction grating that you would use, the proportionality will be constant but the value would change if you were using another grating.

Let's have the chart of the results of r_{β} for the four visible lines in the spectrum of Hydrogen, in line with the values of the wavelengths, of the distance between the energy levels as calculated through the Bohr atomic model, with of the jumps by the linked n-value of the energy levels:

JUMPS	λ	r (between the levels)	r_{β}
6-2	420 nm	1,696 nm	0,3328 nm
5-2	434 nm	1,113 nm	0,2955 nm
4-2	486 nm	0,636 nm	0,2622 nm
3-2	656 nm	0,265 nm	0,3288 nm

Now, let's calculate the surface (S) that is generated from the product of the length of the magnetic vector by the electron displacement:

6-2; S = (0,328 · 1,696) nm = 5,644E-19 m^2

5-2; S = (0,2955 · 1,113) nm = 3,289E-19 m^2

4-2; S = (0,2617 · 0,636) nm = 1,667E-19 m^2

3-2; S = (0,3288 · 0,265) nm = 0,8713E-19 m^2

These area values can be positioned on a scaled line, and you would notice that their spacing strangely resembles the spacings on the spectrum. However, the order of the area values is exactly the reverse of the values of the wavelengths, such that, for example, the value of the difference between A6-2 and A4-2 relates to the spacing between the lines for n (initial) equal to 3 and 5 .

More simply put, after selecting the four wavelengths and the calculation of S, if the S's are set on a scaled line in increasing order, you would be just use the same scale and positions to locate the lines of the wavelengths renumbering by n in a decreasing order. Then, you could compare the distances on your spectrum to the newly renumbered distances on the your calculated scale, and you would find that your measurement on the spectrum and the calculated distances are proportional, within small discrepancies that are understood and tolerated in the field of radiations.

This result is important because it shows that derived formulae that are verified through the Bohr atomic model, can be differentiated and apply to Hydrogen, that the concept of a potential well applies to electron jumps and that the [dF·dr] product, relating to these characterized jumps, goes as far as predicting the spacing between the lines of the linked emitted radiations.

The macro-scale origin of the formulae tells us complementary information, that is about the condition for such righteousness in the results to exist, we also need make sure that the matter in question is homogeneous and that each well offers identical magnetic properties and relative permeability.

But the main point of this rapport between the scaled line showing the surfaces such as the product of the electric, by the magnetic vector and the emitted radiation wavelength, is that it emerges from the process of an incoming frequency or energy by the atom and is kept as a characteristic of the radiated frequencies or energy by the traveling wave and recorded on the spectrum with these characteristics. In summary, the spacing of the lines on the spectrum is a characteristic of the traveling wave that is emitted from the atom, due to the release of extra-loads of energy after the fallback of electrons upon the lower levels of energy. Moreover, the characteristics of the released extra-energy is conditioned to the typical spacing of the lines, that is the quantized emitted energy travels to the threshold of a diffraction grating with the characteristics that are inherited from the process of the initial energy inside the atom.

At this stage, to understand the whole rapport between the product of the magnetic by the electric vector and the emitted wavelengths lines spacing on the spectrum, it is necessary to represent a figure for us. You should see that the spectrum configuration is a central symmetry about I of all the points on the scaled line showing the surfaces in greatness order; then, according to the grating in use, a proportionality factor must apply.

Consequently, before its diffraction through the apertures of the diffraction grating, the represented spectrum through the central symmetry is the expression of an emitted quantum of energy as it is released from the realm of the atom. In other words, the released energy is constituted in a quantum when it has emerged from the central point of focus such as I, whereby I is the only point where the energy balance is equal to the sum of both the energy as generated after the electromagnetic (EM) reaction due to the fall of the unreleased electrons (provided energy) and the radiated energy in a quantum traveling through space with the characteristics from the atom. But this analysis shows us that the distance that we put between the surfaces scale and its image through the symmetry about I, is also to be seen as the distance between the seat of the EM initial reaction and the point where the quanta is formatted to travel through space. Then this distance is a pre-set condition that links to the atom's configuration, and that is specific to both the atom and since the emitted quanta is configured accordingly, to the released extra-load of energy.

Well, this analysis makes it possible to understand why the wavelengths must be added in the reverse order to the scaled line showing the surfaces in greatness order: the symmetry about I imposes it that over the lapse of time taken from the instant an EM reaction occurs (product giving the surfaces) up until the moment energy is constituted in a quanta, all provided and emitted radiations transiting I should be coupled in the way that the total distance that is accomplished from the source point of the EM reaction to the point where the quantum is formatted, remains equal throughout all possible quanta in the same wave packet. As a result, there is a limited number of possible ways that an atom has to release extra-energy, because the pre-figured final energy quantum must bear the characteristics of the line spacings (i.e., the atom's related characteristics) and this way it is liable to the atomic distance between the seat of the caused EM reaction and the point where the quantum is formatted.

We will see now how the previous formulae predict the association in pairs of frequencies through the central symmetry about I. Let's have a chart of the Z values for the following jumps: 6-2; 5-2; 4-2; 3-2.

JUMPS	Z
6-2	2115
5-2	1587
4-2	1053
3-2	1057

You may recall that the surfaces generated are linked to the imposed values of symmetric wavelengths in the detailed way that the EM reaction and related surface value of the 6-2 jump associates the 3-2 jump emitted radiation and the 5-2 jump associates the 4-2 jump emitted radiation.

Moreover, we have seen that the final energy quantum travels with the characteristics that previously linked the values of the squares of each EM reaction's magnetic vector to the value of their related emitted wavelength to the Z value that is linked to the Bohr atomic model and uses the value of the electric vector length.

Consequently, the formatted final quantum carries frequencies that have been paired but remained unchanged in their expressions in the formula. The formula expressing the association of the released frequencies transiting I where they are formatted into a single quantum is also the expression of the frequencies inside the quantum itself, and this formula is:

$$(r1_{\beta} \,.\, r2_{\beta})^2 \cdot F^3 = Z1 \cdot Z2 \quad .$$

This formula reads as the product of the Z values corresponding the jumps of two given frequencies as these which are coupled in the central symmetry about I is equal to the product of the square of their respective magnetic vectors products two by two, multiplying the frequency F at power 3. We are going to see that this frequency is, in fact, equal to the average frequency attached to the average of the wavelengths that are associated two by two.

In effect, let's calculate F for the linked jumps 6-2 and 3-2, using the formula given above:

$$F^3 = (2115 \cdot 1057) / (0.3328E\text{-}9 \cdot 0.3288E\text{-}9)^2 \quad \text{giving out: } F = 5.715E14 \text{ Hz}.$$

The value of the average frequency of the related wavelengths such as 656 nm and 420 nm is this value precisely, and the same findings can be made for the associated jumps of 5-2 and 4-2. Furthermore, the value of that second frequency average is 6.529E14 Hz, and is very close to the former average frequency.

In result, the average of these two frequencies inside the quantum, that is 6E14 Hz is also very close to each of the average frequencies of the coupled wavelengths. These rapports show that the association of the emitted radiations in pairs inside the quantum has an expression through the formula given above and that the pairing (symmetry about I with respect to the conservation of the distance between the seat of the EM reaction and the point of release of the constituted quantum), the atom's configuration (Z values), and the EM reactions (product of magnetic by electric vectors) accounts for the formatting of the released energy in a quantum, the subsequent characteristics of which travel through space at the same time as the energy is radiated away from the atom .

Because of the macro-scale link of these formulae and their thorough application to an electron displacement and emitted radiation in keep, these formulae, issued from my research work, are fundamental in Physics.