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The Gravitational Radius of a Black Hole

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Abstract: It is shown that the equation for the calculation of the Schwarzschild's radius of a black hole does not account the photon's wavelength. If we solve this task in the frame of the space-matter-time unity, we get a new equation for the calculation of black hole's radius that takes into account the wavelength of the electromagnetic radiation (photons). This new mathematical model allows us to calculate gravitational radius of a black hole. These results require us to reconsider the astronomical perspective that we have previously taken.

Keywords: black hole, gravity, photon, space, mass, speed of light, electromagnetic radiation.

Newton's law of gravitation (1687) layed the groundwork for the astronomical ideas for centuries to come [1]. First Michell (1783), then Laplace (1796) predicted the possibility of existence of the stars with a strong enough gravitational field that could retard light and that is why such stars become invisible [2]. Later, they were called the black holes. Einstein's general theory of relativity predicts the existence of the black holes as well [3].

In 1916, Karl Schwarzschild, the German Astronomer and physicist, offers a formula (1) for the calculation of the gravitational radius R_g of a black hole, which originates from classical mechanics [3]. Since then this formula has been used in the astronomic calculations, and the gravitational radius is called Schwarzschild's radius:

$$R_g = \frac{2G \cdot M}{C^2}, \quad (1)$$

where $G = 6.67 \cdot 10^{-11} N \cdot m^2 / kg^2$ is gravitational constant, M is mass of the star, and C is velocity of light.

It is known that as the photon's wavelength is reduced (from the infrared to gamma range), its energy is increased by a factor of 10^{10}. The possibility of a photon to overcome the gravitational force is also increased by the same degree, but it is not taken into account in the formula (1). We have every reason to believe that a mistake has been made when the formula was been derived. What is its kernel, its etiology? The gravitational radius formula (1) was derived by using the mathematical relation in the law of gravitation [3]:

$$F_g = G \cdot \frac{m \cdot M}{R^2} \quad . \quad (2)$$

Here F_g is gravitational force, m is the photon mass, and R is the distance between the centers of the masses of the bodies, which form the gravitation.

In order to find the gravitational radius $R = R_g$ of a star, by which its gravitational field retards light, it is necessary to find a relationship between the gravitational force F_g and the force F_F, which moves the photon. But it is not easily to do when there is no information concerning the electromagnetic structure of the photon. That's why the idea of equality between the energy of the photon E and the potential energy of the gravitational filed E_g was adopted. If we assume that the gravitational force F_g performs the work at a distance, which is equal to the gravitational radius R_g, this work will be equal to [3]:

$$E_g = G \cdot \frac{m \cdot M}{R_g^2} \cdot R_g = G \cdot \frac{m \cdot M}{R_g}. \tag{3}$$

The bonds between the energy of the photon E, its wavelength λ, oscillation frequency ν, and velocity C are determined by the dependencies [4], [5], [6]:

$$E = h \cdot \nu = h \cdot C / \lambda = mC^2, \tag{4}$$

where $h = 6{,}26 \cdot 10^{-34} J \cdot s$ is Planck's constant, and $C = \lambda \cdot \nu$.

It has been assumed that the photon will move in the gravitational field of the star with velocity V, that's why its kinetic energy can be determined by the relation $E_k = mV^2/2$. When $V \Rightarrow C$, we'll have:

$$E_k = mC^2/2. \tag{5}$$

It means that the gravitational field of the star will retard the photon when its potential energy (3) and kinetic energy (5) are equal, i.e.:

$$G \cdot \frac{m \cdot M}{R_g} = \frac{mC^2}{2}. \tag{6}$$

We get formula (1) for the calculation of the gravitational radius, which has been suggested by K. Schwarzschild:

$$R_g = \frac{2G \cdot M}{C^2}. \tag{7}$$

The investigations carried out by us show that the velocity of the center of the photon mass is changed in the interval of its wavelength in such a way that its mean remain constant, and is equal to the velocity of light. We can determine the force, which moves the photon, by of dividing its energy by its wavelength [7], [10]:

$$F_F = \frac{mC^2}{\lambda}. \tag{8}$$

If we equate the gravitational force (2) and the force moving the photon (8), we will have:

$$G \cdot \frac{m \cdot M}{R_g^2} = \frac{mC^2}{\lambda}. \tag{9}$$

It is clear that in order to convert the equation of the forces (9) into the equation of energies (6) it is necessary to reduce the denominator of the left-hand part by the gravitation radius R_g and to reduce the denominator of the right-hand side by the wavelength λ of the photon. Actually it means that it is necessary to equate the gravitational radius to the wavelength of the photon. Certainly, it is impossible to do it, but Schwarzschild did it, and his followers did not notice this error. Thus, we have found the reason for the absence of the wavelength of the photon in formula (1) in order to calculate the gravitational radius of the black hole. It is the result of the mistake that Schwarzschild made when he derived this formula.

In order to determine the gravitational radius of a black hole it is necessary to use the equality between the gravitational force and the force that moves the photon, but not the equality of energies. Now we are faced with a difficult problem: we should be able to find a mathematical model of this force, which moves the photon.

Since the photon in motion has mass, it has the center of mass as well. As the photon is polarized, it moves in the plane along the wave's path in order to provide the photon with its wave-like properties. With which equations is it possible to describe the wave's movement about the photon's center of mass?

The answer for this question can be given by the space – matter – time unity axiom [6], [7], [10]. The space – matter – time unity is realized only in mathematical models where the coordinates of the moving object (i.e., the center of a photon's mass) are a function of time [6], [7], [10].

It is clear that Louis de Broglie's wave equation:

$$y = A\sin 2\pi(\nu t - x/\lambda) \tag{10}$$

cannot play this role because it is single, and coordinate x and time t are the independent variables in it [6]. As time flows in the process of the changing of the coordinate, the coordinate and time are the dependent variables in reality. Definite time corresponds to each point in space, in which the photon's center of mass is. Since the center of mass belongs to the "material" photon, this state is called the space-matter-time unity. Thus, the equation of Louis de Broglie (10) does not correspond to the space-matter-time unity, which is why it cannot describe the wave's movement about the photon's center of mass within the framework of this unity. A comprehensive analysis of this problem has been carried out by us and it has shown that only the equations of the shortened cycloid can play this role [4], [5], [6], [7]. The photon's center of mass makes the most economical oscillations (from an energy the point of view) only when the wave-length λ of one oscillation is equal to the radius r of its rotation [4], [5], [6], [7];

$$\lambda = r. \tag{11}$$

Taking into consideration the aforementioned facts, the following equations of the photon's center of mass have been obtained [4], [5], [6], [10]:

$$x = Ct + \frac{r}{2\pi}\sin 6\omega t; \tag{12}$$

$$y = \frac{r}{2\pi}\cos 6\omega t, \tag{13}$$

where ω is the angular frequency of rotation of the photon: $\alpha = 60^0$ is the angle, which characterizes the structure of the photon.

The projections of velocity of the photon's center of mass on the immovable coordinate axes XOY will be determined by the dependencies:

$$V_x = C + r\nu\cos 6\omega t; \tag{14}$$

$$V_y = -r\nu \sin 6\omega t. \qquad (15)$$

The function of velocity of the photon's center of mass will be:

$$V = \sqrt{V_x^2 + V_y^2} = 2C \cdot \cos 3\omega t. \qquad (16)$$

Figure 1 is a diagram of velocity of the photon's center of mass, which shows that the average value remains constant and is equal C.

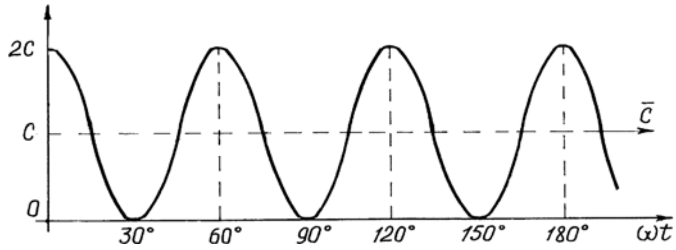

Fig. 1. Diagram of the velocity of the photon's center of mass

It is known that the work is performed by the tangential component of force. That's why we should know the tangential acceleration of the photon's center of mass. It will be:

$$a_\tau = \frac{dV}{dt} = -6C\omega \sin 3\omega t. \qquad (17)$$

Then the force moving the photon's center of mass will be:

$$F_F = m \cdot a_\tau = -6Cm\omega \cdot \sin 3\omega t = -2\pi \cdot \frac{mC^2}{\lambda} \cdot \sin 3\omega t. \qquad (18)$$

The force has its maximal value when $\sin \omega t = -1$; that's why finally we'll have:

$$F_F = 2\pi \cdot \frac{mC^2}{\lambda}. \qquad (19)$$

If force F_F, which moves the photon, is equal to gravitational force F_g, then the force of the gravitational field retards the photon:

$$G \cdot \frac{m \cdot M}{R_g^2} = 2\pi \cdot \frac{mC^2}{\lambda}. \qquad (20)$$

We have the gravitational radius:

$$R_g = \frac{1}{C}\sqrt{\frac{G \cdot M \cdot \lambda}{2\pi}}. \qquad (21)$$

Thus, we have the formula for the calculation of the gravitational radius of a black hole, which takes into consideration the wavelength of the electromagnetic radiation.

Then the force F_F, which moves the photon with the wavelength $\lambda = 0{,}65 \cdot 10^{-6}$ m and the $velocity \cdot 2.998 \cdot 10^8 \, m/s$, will be [6]:

$$F_F = 2\pi \cdot \frac{h \cdot \nu}{\lambda} = 2\pi \cdot \frac{h \cdot C}{\lambda^2} = 2\pi \cdot \frac{6.26 \cdot 10^{-34} \cdot 2.998 \cdot 10^8}{(0.65 \cdot 10^{-6})^2} = 2.79 \cdot 10^{-12}, N \qquad (22)$$

Assuming that mass of the Sun is $M = 2 \cdot 10^{30} \, kg$, its radius is $R = 6.96 \cdot 10^8 \, m$, the gravitational constant $G = 6.67 \cdot 10^{-11} \, N \cdot m^2 / kg^2$, and mass of the photon is designated as being "m" and takes into account equation (4), then we can determine the force of gravitation of the Sun F_g, which influences the passing photon according to the formula (1), (7):

$$\begin{aligned} F_g &= G \cdot \frac{m \cdot M}{R^2} = G \frac{h \cdot M}{\lambda \cdot C \cdot R^2} = 6.67 \cdot 10^{-11} \times \\ &\times \frac{6.26 \cdot 10^{-34} \cdot 2.0 \cdot 10^{30}}{0.65 \cdot 10^{-6} \cdot 2.998 \cdot 10^8 \cdot (6.96 \cdot 10^8)^2} = 0.88 \cdot 10^{-33} \, N. \end{aligned} \qquad (23)$$

When the photon passes the Sun, tangentially, the angle of deviation of the photon from the straight-line motion will be equal to $tg\alpha = F_g / F_F = 0.31 \cdot 10^{-21}$. If a photon with the wavelength of $\lambda = 0.65 \cdot 10^{-6} \, m$ passes the Sun along the straight line, which is parallel to the line connecting the centers of Sun and the Earth, the value of its deviation ΔS from the straight-line motion in the vicinity of the Earth will be [7], [10]:

$$\Delta S = L \cdot tg\alpha = 1.51 \cdot 10^{11} \cdot 0.31 \cdot 10^{-21} = 0.48 \cdot 10^{-10} \, m, \qquad (24)$$

where $L = 1.51 \cdot 10^{11} \, m$ is the distance from the Earth to the Sun. At present time, science has no devices, which can register the value ΔS.

Now gravitational radius R_g of the Sun, by which it becomes a black hole, is determined according to the formula (1), which does not take into account the wavelength of the photon [2], [3]:

$$R_g = \frac{2G \cdot M}{C^2} = \frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 2 \cdot 10^{30}}{(2.998 \cdot 10^8)^2} = 2.97 \cdot 10^3 \, m. \qquad (25)$$

Let us determine gravitational radii of the Sun for the infrared, visible, and gamma photons with the following wavelengths (respectively): $\lambda_r = 1.0 \cdot 10^{-3} \, m$, $\lambda_l = 0.65 \cdot 10^{-6} \, m$, and $\lambda_g = 1.0 \cdot 10^{-14} \, m.$

$$R_{gr} = \frac{1}{C} \sqrt{\frac{G \cdot M \cdot \lambda_r}{2\pi}} = \frac{1}{2.998 \cdot 10^8} \sqrt{\frac{6.67 \cdot 10^{-11} \cdot 2.0 \cdot 10^{30} \cdot 1.0 \cdot 10^{-3}}{2 \cdot 3.14}} = 4.39 \cdot m. \qquad (26)$$

$$R_{gl} = \frac{1}{C} \sqrt{\frac{G \cdot M \cdot \lambda_l}{2\pi}} = \frac{1}{2.998 \cdot 10^8} \sqrt{\frac{6.67 \cdot 10^{-11} \cdot 2.0 \cdot 10^{30} \cdot 0.65 \cdot 10^{-6}}{2 \cdot 3.14}} = 0.012 m. \qquad (27)$$

$$R_{gg} = \frac{1}{C}\sqrt{\frac{G \cdot M \cdot \lambda_g}{2\pi}} = \frac{1}{2.998 \cdot 10^8}\sqrt{\frac{6.67 \cdot 10^{-11} \cdot 2.0 \cdot 10^{30} \cdot 1.0 \cdot 10^{-14}}{2 \cdot 3.14}} = 1.39 \cdot 10^{-5} m. \quad (28)$$

Under usual conditions, the density ρ of the Sun is $1.4 kg/m^3$ [1]. After compression, the density of the Sun will depend on the gravitational radius, which is determined according to the formulas (25), (26), (27) and (28), respectively:

$$\rho_o = \frac{3M}{4\pi \cdot R_g^3} = \frac{3 \cdot 2 \cdot 10^{30}}{4 \cdot 3.14 \cdot (2.97 \cdot 10^3)^3} = 1.82 \cdot 10^{19} kg/m^3. \quad (29)$$

$$\rho_r = \frac{3M}{4\pi \cdot R_{gr}^3} = \frac{3 \cdot 2 \cdot 10^{30}}{4 \cdot 3.14 \cdot (4.39)^3} = 5.65 \cdot 10^{27} kg/m^3. \quad (30)$$

$$\rho_l = \frac{3M}{4\pi \cdot R_{gl}^3} = \frac{3 \cdot 2 \cdot 10^{30}}{4 \cdot 3.14 \cdot (0.12)^3} = 5.53 \cdot 10^{32} kg/m^3. \quad (31)$$

$$\rho_g = \frac{3M}{4\pi \cdot R_{gg}^3} = \frac{3 \cdot 2 \cdot 10^{30}}{4 \cdot 3.14 \cdot (1.39 \cdot 10^{-5})^3} = 0.178 \cdot 10^{45} kg/m^3. \quad (32)$$

Remember that the density of nuclei of the atoms is determined by the value $(1.2 - 2.4) \cdot 10^{17} kg/m^3$ [8].

It is clear that if the Sun were compressed to the gravitational radius $R_{gr} = 4.39m$ (26), its gravitational field would retard only the radiation in the far infrared range of the spectrum. Electromagnetic radiation with smaller wavelengths will penetrate more easily. In order to retard the photons of all frequencies, the gravitational radius of the Sun should be $R_{gg} = 1.39 \cdot 10^{-5} m$ (28), which is hardly possible because, in this case, the density of the substance of the Sun would be 10^{28} times greater than density of an atom's nuclei [7], [10].

CONCLUSION

Thus, an error in the determination of the gravitational radius of a black hole according to formula (1), which does not take into account the wavelength of the electromagnetic radiation, is 10^8 times (based upon calculations using the gravitational radius of our Sun), but the astronomers do not know it yet [7], [10].

If objects with a gravitational field that can retard electromagnetic radiation exist in the nature, then not all of them can be black. Their colors should be changed in compliance with the change of the wavelength of the photons that are not retarded by these objects. The photons from the infrared range of the spectrum will be the first to be retarded, as their gravitational radius is reduced, followed by the visible, ultraviolet, roentgen, and gamma range photons. The hole becomes a black one only in a case where the gravitational radius corresponds to the gamma photon with the minimal wavelength.

American astrophysicists have experimental results that show that black hole radiate in the ultraviolet, roentgen, and gamma photons [11]. This theory predicts such results. It also allows one to elaborate on the results of any astronomical observations.

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