Possible Existence of Tachyon Field Cancellation of ZPF Induced Gravitational Field in Empty Space

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Abstract: Quantum electrodynamics shows that empty space is filled with the zero-point fluctuations (ZPF) of electromagnetic energy field. In accordance with the theory of relativity, the ZPF is assumed to be a gravitational source and the radius of the universe is expected to be smaller than the size of the atomic nucleus. In this paper, the author attempts to solve this riddle by introducing tachyon field which consists of virtual faster-than-light (FTL) photons created out of the ZPF background.

Keywords: tachyon, faster-than-light, zero-point energy

Symbols:

ψ: wave function of the moving particle
α: proper acceleration of the particle
c: light speed
h: Plank’s constant divided by $2\pi$
ρ_E: energy density of the ZPF
ν: velocity of the particle in FTL state
m_0: absolute value of the tachyon’s rest mass
T: probability for the particle exceeding the light speed
ω: ZPF cutoff frequency
l_p: Plank length
G_{μν}: Einstein tensor
g_{μν}: metric tensor
T_{μν}: energy-momentum tensor of matter
λ: cosmological constant
G: gravitational constant
R: radius of the curvature of space
Introduction

Quantum electrodynamics shows that empty space is filled with the zero-point fluctuations (ZPF) of electromagnetic energy field which has spectral energy density given by

$$\rho(\omega) d\omega = \frac{\hbar}{2 \Omega^3 c^3} d\omega,$$  \hspace{1cm} (1)

where $\rho(\omega)$ is energy density of the ZPF, $\omega$ is its angular frequency, $\hbar$ is a Plank’s constant divided by $2\pi$ and $c$ is the light speed. The total, frequency-integrating energy density of the ZPF becomes

$$\rho_E = \int_0^\omega \frac{\hbar}{2 \Omega^3 c^3} d\omega = \frac{\hbar \Omega^4}{8 \Omega^2 c^3},$$  \hspace{1cm} (2)

where $\omega_\Omega$ is the ZPF cutoff frequency.

In accordance with the theory of relativity, $\rho_E$ is assumed to be a gravitational source and it contributes to the curvature of space which is described by the gravitational field equation as

$$G_{\mu\nu} - \kappa g_{\mu\nu} = -k(T_{\mu\nu} + \frac{1}{2} g_{\mu\nu} T),$$  \hspace{1cm} (3)

where $G_{\mu\nu}$ is the Einstein tensor, $g_{\mu\nu}$ is a metric tensor, $\kappa$ is a cosmological constant, $T_{\mu\nu}$ is the energy-momentum tensor of matter and $k = 8\pi G / c^2$.

From which the radius of curvature of space $R$ can be approximated by

$$\frac{2}{R^2} = \frac{k \rho_E}{2 c^2},$$  \hspace{1cm} (4)

under weak gravitational field if $\kappa = 0$.

Supposing that $\omega_\Omega$ is on the order of the Plank frequency $\omega_p$ given by

$$\omega_p = \sqrt{c^5 / \hbar G} \approx 2 \times 10^{43} \text{ s},$$  \hspace{1cm} (5)

the radius of the universe can be expected to be smaller than the size of the atomic nucleus if the ZPF were real. In this paper, I will attempt to solve this riddle by introducing a tachyon field which consists of the virtual faster-than-light (FTL) photons created out of the ZPF background.

Possibility of Tachyons Created out of Empty Space

The author presented the possibility of FTL phenomena in his scientific paper that FTL speed can be permitted for highly accelerated elementary particles by quantum tunneling effect, the derivation of which is described as follows.

The time dependent wave function of the particle can be written as

\begin{verbatim}
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\[ i\hbar \frac{\partial \psi}{\partial t} = E\psi, \quad (6) \]

where \( \psi \) is a wave function of the particle, \( E \) is its energy and \( \hbar \) is the Planck’s constant divided by \( 2\pi \). By the special relativity theory, \( E \) is given by

\[ E = \sqrt{m_0^2 c^4 + p^2 c^2}, \quad (7) \]

where \( m_0 \) is a proper mass of the particle, \( c \) is the light speed and \( p \) is the momentum of the particle shown as:

\[ p = \frac{m_0 v}{(1 - v^2 / c^2)^{1/2}}. \quad (8) \]

Eqs. (6), (7) and (8) together give

\[ \frac{\partial \psi}{\partial t} = -i \frac{m_0 c^3}{\hbar} \frac{\psi}{(c^2 - v^2)^{1/2}}. \quad (9) \]

By using the chain rule of the derivatives shown as \( \frac{\partial \psi}{\partial t} = \frac{d\psi}{dv} \frac{dv}{dt} \) and the following formula for the accelerated particle given by

\[ \frac{dv}{dt} = \Box (1 - v^2 / c^2)^{3/2}, \quad (10) \]

Eq. (9) can be rewritten as

\[ \frac{d\psi}{dv} = -i \frac{m_0 c^6}{\hbar} \frac{\psi}{(c^2 - v^2)^{3/2}}, \quad (11) \]

where \( \alpha \) is a proper acceleration of the particle.

By the following formula of integral calculus shown as

\[ \int \frac{dv}{(c^2 - v^2)^{3/2}} = \frac{v}{2c^2(c^2 - v^2)} + \frac{1}{4c^3} \log \left( \frac{c + v}{c - v} \right), \quad (v \leq c) \]

\[ = -\frac{v}{2c^2(v^2 - c^2)} + \frac{1}{4c^3} \log \left( \frac{v - c}{v + c} \right), \quad (v > c) \quad (12) \]

the wave function \( \psi \) below the light speed becomes

\[ \psi = C_0 \exp \left[ -i \frac{m_0 c^6}{\hbar} \left( \frac{v}{2c^2(c^2 - v^2)} + \frac{1}{4c^3} \log \left( \frac{c + v}{c - v} \right) \right) \right], \quad (13) \]
where $C_0$ is an arbitrary constant.

Above the light speed the particle is required to have an imaginary rest mass from the theory of special relativity, the wave function $\psi$, of the particle travelling in a tachyonic mode can be given as follows by inserting the imaginary proper mass $im_0$ to the term $m_0$ in Eq. (13),

$$\psi = C_0 \exp \left[ \frac{m_0 c^3}{\hbar} \left( \frac{v_s}{2c^2 (v_s^2 - c^2)} + \frac{1}{4c^3} \log \left( \frac{v_s - c}{v_s + c} \right) \right) \right], \quad (14)$$

where $m_*$ is an absolute value of the FTL particle's rest mass and $v_*$ is the velocity of the FTL particle.

Thus the probability of the virtual photon which exceeds the light speed can be given by

$$T = \left| \frac{\psi_s}{\psi} \right|^2 = \exp \left[ \frac{m_* c^3}{\hbar} \left( \frac{1}{2} \log \left( \frac{v_* - c}{v_* + c} \right) - \frac{cv_*}{v_*^2 - c^2} \right) \right]. \quad (15)$$

Hence it is seen that higher accelerated particles have the probability to penetrate the light barrier as shown in Fig. 1 below.

**Fig. 1** Quantum tunneling of the particle through the light barrier.

**Mass Density of the Virtual Tachyon in Empty Space**

In empty space, virtual particles, most of which are low energy photons, are created out of the ZPF background. Supposing that the tunneling photon through the light barrier satisfies the energy conservation law shown as
FTL photons have an imaginary proper mass which can be expressed as

\[ m_c = \frac{\hbar \omega}{c^2 \sqrt{1 - v_c^2 / c^2}}, \]  

(17)

where \( \hbar \omega \) is the energy of the virtual photon created out of ZPF background. Since the FTL particle follows the uncertainty principle for the tachyon given by

\[ \Delta P \cdot \Delta t = \frac{\hbar}{v_c - c}, \]  

(18)

From which FTL velocity of the photon can be roughly estimated as

\[ v_c = c \left(1 - \sqrt{\frac{1}{\hbar \omega \Delta t}}\right). \]  

(20)

By using the uncertainty relation \( \Delta E \cdot \Delta t = \hbar \), FTL velocity of the virtual photon, which has the energy \( \Delta E = \hbar \omega \), can be approximated as

\[ v_c = 2c. \]  

(21)

Substituting this value into Eq. (17), the absolute mass of the FTL photon becomes

\[ m_c = \frac{\sqrt{3}}{c^2} \hbar \omega. \]  

(22)

If the virtual photon is created within a region which size is almost equal to the Plank length \( l_p \), by using \( \Delta P \cdot l_p = \hbar \) and \( m = \hbar \omega / c^2 \), its acceleration is roughly estimated as follows:

\[ \ddot{a} \approx \frac{1}{m} \frac{\ddot{P}}{\ddot{t}} = \frac{c^2}{l_p}. \]  

(23)

From which the probability of the photon transformed into tachyonic state becomes
\[ T_p \approx \exp \left( -\frac{\gamma l_p}{c^2} \omega \right) \text{,} \quad (24) \]

where \( \gamma = \sqrt{3 \left( \frac{\log 3}{2} + \frac{2}{3} \right)} \).

Supposing that almost all of virtual particles created out in empty space are photons, the absolute mass density for virtual tachyons can be estimated as

\[ \rho_{v} \approx \frac{3\hbar}{\pi c^2} T_p d \omega \text{,} \quad (25) \]

where the integral term is expressed as:

\[
\int_{0}^{\infty} \exp \left( -\frac{\gamma l_p}{c^2} \omega \right) \omega^3 d\omega = \frac{6c^4}{\gamma^4 l_p^4} \left( \frac{6c^4}{\gamma^4 l_p^4} + \frac{6c^3 \omega_p}{\gamma^3 l_p^3} + \frac{3c^2 \omega_p^2}{\gamma^2 l_p^2} + \frac{c \omega_p^3}{\gamma l_p} \right) \exp \left( -\frac{\gamma l_p \omega_p}{c} \right). \quad (26)
\]

From this equation, virtual FTL photons created in empty space have non-zero imaginary mass density at the Plank frequency as shown in Fig. 2 below.

![Fig. 2](http://www.journaloftheoretics.com/Articles/2-4/zpf-pub.htm(6 of 9)[8/5/2000 5:35:14 PM])

**Fig. 2** Calculated result of \( \rho_{v} \) vs. \( \omega_c \) (calculated by Mathematica 3.0).

**Repulsive Force Generated by the Tachyon Field**

From Eq. (3), the gravitational equation containing the universal constant can be approximated as

\[
\frac{\omega^4}{\omega_0^4} = \frac{\rho_{v}}{\rho_{0}} \left( \frac{\omega}{\omega_0} \right) \text{,}
\]

where \( \rho_{v} \) is the absolute mass density of virtual tachyons, \( \rho_{0} \) is the mass density of photons, and \( \omega_0 \) is the Plank frequency.
\[-\frac{2}{R^2} + \lambda = -\frac{k}{2} \rho_m, \quad (27)\]

where \( \rho_m = \frac{\rho_k}{c^2} = \frac{\hbar}{8\pi^2 c^3} \omega^4 \).

Supposing that the Newtonian law of gravitation can be applied for particles which have an imaginary mass \( \text{im}_m \), the repulsive force \( f \) generated between them, the separation distance \( r \), becomes
\[ f = G \frac{\text{im}_m \cdot \text{im}_m}{r^2} = -G \frac{m^2}{r^2}. \quad (28) \]

If the FTL photons are uniformly distributed and infinitely extended in a space, the repulsive force generated by them makes the cosmological constant to have \[ \lambda = -\frac{k}{2} \rho_m. \quad (29) \]

Combined this relation with Eq. (27), we have
\[ -\frac{2}{R^2} = -\frac{k}{2} \rho_m (1 - \rho_r / \rho_m). \quad (30) \]

By Eqs. (26) and (27), the ratio \( \Box / \Box_m \) vs. \( \omega_c \) can be calculated as shown in Fig. 3.

From this figure, it is seen that \( \rho_r / \rho_m \) becomes unity at almost \( 10^{41} / s \), which suggests that the gravitational field due to the ZPF can be canceled by the virtual FTL photons created out of the ZPF background.

![Fig. 3](http://www.journaloftheoretics.com/Articles/2-4/zpf-pub.htm)  
Calculated result of the ratio \( \Box / \Box_m \) vs. \( \omega_c \) (calculated by Mathematica 3.0).
Conclusion

In this paper, I have attempted to solve the problem where the radius of the universe is expected to be smaller than the size of the atomic nucleus by the ZPF induced gravitational field, if the ZPF cutoff frequency is on the order of the Plank frequency. By the theoretical analysis, it is shown that the gravitational field due to the ZPF can be canceled by the tachyon field created out of the ZPF background.

Endnote

By supposing that \( L \geq l_p \) must be satisfied for virtual photons, where \( L \) is a traveling distance of them in a tachyonic state, the upper limit of tachyons created from ZPF background can be estimated. From the uncertainty of energy and momentum, we obtain \( L = c/\omega \). Then the upper frequency can be given as \( \omega = c/l_p \), which yields the Plank frequency.

References
